11.2 Velocity and acceleration analysis

Name: _____ Class: _____ Date: _____ **Total: 17 marks**

KEY WORDS velocity polygon • acceleration polygon • instant centre • Coriolis component

Objective

Find the velocities and accelerations of a slider-crank from its polygons, the angular velocity of the rod, and the Coriolis component of a block sliding on a turning arm.

1 Key knowledge and worked examples

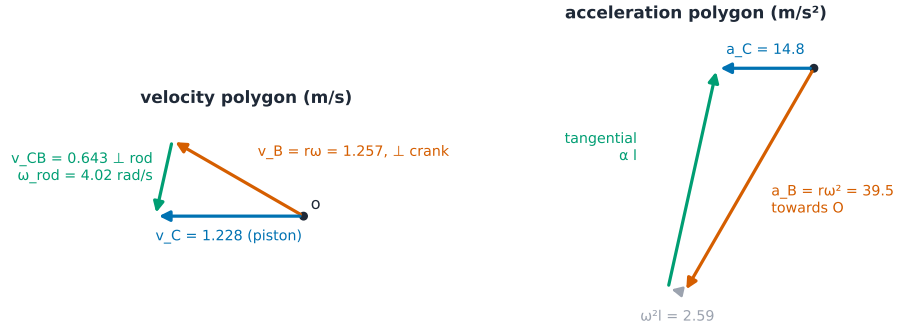
■ Polygons and the Coriolis term

- On one rigid link, $\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{CB}$: the relative velocity is across the link (**velocity polygon**).
- Acceleration adds $\omega^2 r$ along the link and αr across it (**acceleration polygon**).
- Slider-crank results: $v_C = r\omega \sin(\theta+\phi)/\cos\phi$, $\omega_3 = r\omega\cos\theta/(l\cos\phi)$, and $a_C \approx r\omega^2(\cos\theta + \cos 2\theta/n)$, $n = l/r$.
- **Coriolis component** $2\omega v_{rel}$ for sliding on a rotating link; an **instant centre** gives a check.

■ Worked example. Velocities at 60°

$r = 40$ mm, $l = 160$ mm, 300 rpm, $\theta = 60°$, $\phi = 12.5°$.

- $\omega = 2\pi N/60 = 2\pi \times 300/60 = \mathbf{31.42}$ rad/s; $v_B = r\omega = \mathbf{1.257}$ m/s.
- Polygon: $v_C = \mathbf{1.228}$ m/s, $v_{CB} = 0.643$ m/s, so $\omega_3 = v_{CB}/l = \mathbf{4.02}$ rad/s.
- $a_B = r\omega^2 = 39.5$ m/s^2; closing the acceleration polygon gives $a_C = \mathbf{14.8}$ m/s^2.



2 Practice

A slider-crank has $r = 50$ mm and $l = 200$ mm and turns at 600 rpm.

2.1 Find ω (two decimals, rad/s) and the crank-pin speed v_B (three decimals, m/s). [2]

2.2 At $\theta = 45°$ ($\phi = 10.18°$) find the piston speed v_C (three decimals, m/s), the rod's angular velocity ω_3 (two decimals, rad/s) and v_{CB}. [5]

2.3 A block slides outwards at 0.4 m/s along a slotted arm turning at 8 rad/s. Find the Coriolis acceleration and its direction. [3]

3 Exam-style questions

3.1 For the slider-crank of 2.2 at $\theta = 45°$: (a) find the crank-pin acceleration and the rod's centripetal acceleration (one decimal, m/s^2); (b) estimate the piston acceleration with the approximate formula (one decimal, m/s^2) and give its direction; (c) find the inertia force on a 1.2 kg piston (nearest N) and say why the approximation is good here. [7]

Solutions

2.1

- $\omega = 2\pi N/60 = 2\pi \times 600/60 = \mathbf{62.83}$ rad/s.
- $v_B = r\omega = 0.05 \times 62.83 = \mathbf{3.142}$ m/s.

2.2

- $v_C = r\omega \sin(\theta+\phi)/\cos\phi = 3.142 \sin 55.18°/\cos 10.18° = \mathbf{2.620}$ m/s.
- $\omega_3 = r\omega\cos\theta/(l\cos\phi) = 3.142\cos 45°/(0.2\cos 10.18°) = \mathbf{11.28}$ rad/s.
- $v_{CB} = \omega_3 l = 11.28 \times 0.2 = \mathbf{2.257}$ m/s, across the rod.

2.3

- $a_{Cor} = 2\omega v_{rel} = 2 \times 8 \times 0.4 = \mathbf{6.4}$ m/s^2.
- Perpendicular to the arm, towards the side the arm is turning (the sliding velocity rotated 90° with ω).

3.1

- (a) $a_B = r\omega^2 = 0.05 \times 62.83^2 = \mathbf{197.4}$ m/s^2, towards O.
- Rod centripetal $\omega_3^2 l = 11.28^2 \times 0.2 = \mathbf{25.5}$ m/s^2, from C towards B.
- (b) $n = l/r = 4$; $a_C \approx r\omega^2(\cos\theta + \cos 2\theta/n) = 197.4(0.7071 + 0) = \mathbf{139.6}$ m/s^2.
- Directed towards the crank: past outer dead centre the piston is speeding up as it moves in.
- (c) $F = ma_C = 1.2 \times 139.6 = \mathbf{168}$ N.
- The dropped terms are of order $(r/l)^3$; with $n = 4$ the exact polygon gives 140.4 m/s^2, 0.6% more.