11.1 Mechanisms and kinematic pairs

Name: _____ Class: _____ Date: _____ **Total: 17 marks**

KEY WORDS mechanism • kinematic pair • degrees of freedom • Gruebler's equation • Grashof condition • inversion

Objective

Count a mechanism's degrees of freedom, place a slider-crank by loop closure, and decide with the Grashof condition what a four-bar chain can do.

1 Key knowledge and worked examples

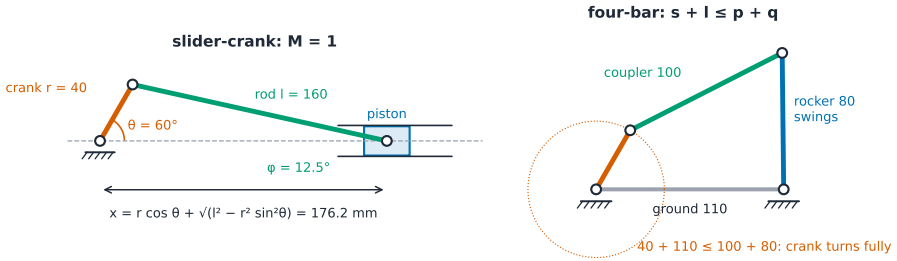
■ Links, pairs and mobility

- A **mechanism** is **links** joined by **kinematic pairs**: pins and sliders allow one motion, cam or gear contact two.
- **Degrees of freedom** by **Gruebler's equation**: $M = 3(n-1) - 2j_1 - j_2$, ground counted as a link.
- Slider-crank: $\sin\phi = r\sin\theta/l$, $x = r\cos\theta + \sqrt{l^2 - r^2\sin^2\theta}$; stroke $2r$.
- **Grashof condition** $s + l \le p + q$: some link turns fully. Shortest link as crank: crank-rocker; as ground: double-crank; as coupler: double-rocker (the **inversions**).

■ Worked example. The dosing pump's position

$r = 40$ mm, $l = 160$ mm, $\theta = 60°$.

- $\sin\phi = r\sin\theta/l = 40\sin 60°/160$, so $\phi = \mathbf{12.5}°$.
- $x = r\cos\theta + \sqrt{l^2 - r^2\sin^2\theta} = 20 + 156.2 = \mathbf{176.2}$ mm; stroke $2r = 80$ mm.
- 4 links, 4 lower pairs: $M = 3(n-1) - 2j_1 = 3\times 3 - 2\times 4 = \mathbf{1}$.



2 Practice

2.1 A slider-crank has $r = 50$ mm and $l = 200$ mm. At $\theta = 45°$ find the rod angle ϕ (two decimals, degrees) and the piston position x (one decimal, mm). State the stroke. [4]

2.2 Find M for (a) a six-bar chain of 6 links and 7 pins, and (b) a mechanism of 5 links with 5 pins and one cam contact. [3]

2.3 A chain has links of 30, 90, 70 and 100 mm. Is it Grashof? Name the mechanism when the 30 mm link is the ground. [3]

3 Exam-style questions

3.1 A four-bar has crank 50 mm, coupler 120 mm, rocker 90 mm and ground 130 mm, with the rocker pivot 130 mm to the right of the crank pivot. (a) Show it is a crank-rocker. (b) At $\theta_2 = 90^\circ$ find the distance from the crank pin to the rocker pivot (one decimal, mm), and by the cosine rule the coupler angle θ_3 and rocker angle θ_4 (one decimal, degrees, open assembly). [7]

Solutions

2.1

- $\sin\phi = r\sin\theta/l = 50\sin 45°/200 = 0.1768$, so $\phi = \mathbf{10.18}°$.
- $x = r\cos\theta + \sqrt{l^2 - r^2\sin^2\theta} = 35.4 + 196.9 = \mathbf{232.2}$ mm.
- Stroke $= 2r = \mathbf{100}$ mm.

2.2

- (a) $M = 3(n-1) - 2j_1 = 3\times 5 - 2\times 7 = \mathbf{1}$.
- (b) $M = 3(n-1) - 2j_1 - j_2 = 3\times 4 - 2\times 5 - 1 = \mathbf{1}$: the cam is a two-freedom pair, so it removes one.

2.3

- $s + l = 30 + 100 = 130 \le p + q = 90 + 70 = 160$: Grashof.
- With the shortest link fixed, both neighbours turn fully: a **double-crank** (drag link).

3.1

- (a) $s + l = 50 + 130 = 180 \le 120 + 90 = 210$: Grashof, and the shortest link is the crank, so it is a crank-rocker.
- (b) Crank pin $B = (0, 50)$, rocker pivot $O_4 = (130, 0)$: $BO_4 = \sqrt{130^2 + 50^2} = \mathbf{139.3}$ mm, at $-21.0°$ from B.
- Cosine rule in triangle BCO_4: $\cos\alpha = (b^2 + BO_4^2 - c^2)/(2b\,BO_4) = 0.7688$, so $\alpha = 39.8°$.
- $\theta_3 = -21.0° + 39.8° = \mathbf{18.7}°$.
- $C = B + 120(\cos 18.7°, \sin 18.7°) = (113.7, 88.5)$ mm, so $\theta_4 = \text{atan2}(88.5, 113.7 - 130) = \mathbf{100.5}°$.