10.5 Heat exchangers

Name: _____ Class: _____ Date: _____ **Total: 18 marks**

KEY WORDS log mean temperature difference • overall heat transfer coefficient • effectiveness–NTU method • counterflow • parallel flow • fouling resistance

Objective

Size a heat exchanger by the log mean temperature difference, compare counterflow with parallel flow, and rate a fouled unit by the effectiveness–NTU method.

1 Key knowledge and worked examples

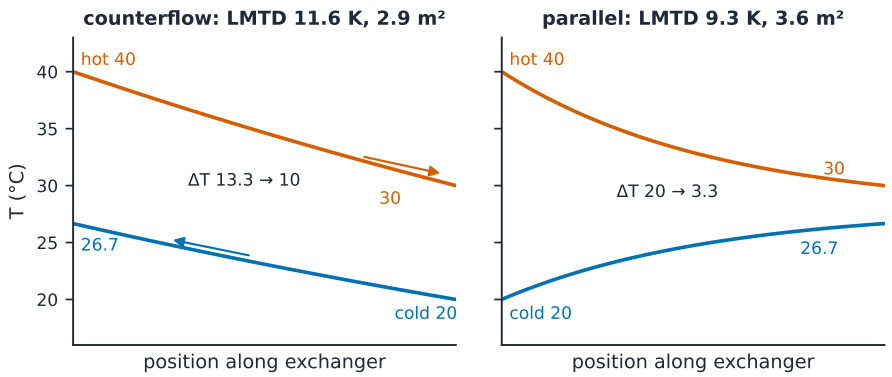
Two methods

- Energy balance: $Q = \dot{m}_h c_h \Delta T_h = \dot{m}_c c_c \Delta T_c$.
- **Log mean temperature difference** : $\Delta T_{lm} = (\Delta T_1 - \Delta T_2)/\ln(\Delta T_1/\Delta T_2)$ over the two ends; $Q = U A \Delta T_{lm}$, with the **overall heat transfer coefficient** U.
- **Effectiveness–NTU method** – (outlets unknown): $NTU = UA/C_{min}$, $C_r = C_{min}/C_{max}$; for **counterflow** $\varepsilon = \dfrac{1 - e^{-NTU(1-C_r)}}{1 - C_r e^{-NTU(1-C_r)}}$, and $Q = \varepsilon C_{min}(T_{h,in} - T_{c,in})$.
- **Fouling resistance** : $1/U = 1/U_{clean} + R_f$.

Worked example. Sizing by LMTD

Loop water 2.0 kg/s, 40 → 30 °C; plant water 3.0 kg/s in at 20 °C.

- $Q = \dot{m} c \Delta T = 2.0 \times 4180 \times 10 = \mathbf{83.6}$ kW; plant water leaves at 26.7 °C.
- Counterflow ends 13.3 and 10 K: $\Delta T_{lm} = \mathbf{11.6}$ K.
- $A = Q/(U \Delta T_{lm}) = 83\,600/(2500 \times 11.6) = \mathbf{2.89}$ m²; **parallel flow** needs 3.59 m².



2 Practice

Oil $c = 2100$ J/(kg · K); water $c = 4180$ J/(kg · K).

2.1 Oil at 1.5 kg/s is cooled from 90 to 60 °C by water at 2.0 kg/s entering at 20 °C. Find the duty (one decimal, kW) and the water outlet temperature (one decimal, °C). [3]

2.2 For counterflow, find ΔT_{lm} (two decimals, K) and the area at $U = 400$ W/(m² · K) (two decimals, m²). [4]

2.3 Repeat 2.2 for parallel flow and say which arrangement is smaller. [3]

3 Exam-style questions

3.1 The counterflow unit of 2.2 ($A = 4.85$ m^2) fouls with $R_f = 0.0005$ m$^2 \cdot$ K/W. Find the new U (one decimal), C_{min}, C_r (four decimals), NTU (three decimals) and ε (three decimals). Find the duty now (one decimal, kW) and the oil outlet temperature (one decimal, °C), and say why LMTD alone cannot answer this directly. [8]

Solutions

2.1

- $Q = \dot{m}_h c_h \Delta T_h = 1.5 \times 2100 \times 30 = \mathbf{94.5}$ kW.
- $T_{c,out} = 20 + Q/(\dot{m}_c c_c) = 20 + 94\,500/8360 = \mathbf{31.3}$ °C.

2.2

- Counterflow ends: $90 - 31.3 = 58.7$ K and $60 - 20 = 40$ K.
- $\Delta T_{lm} = (58.7 - 40)/\ln(58.7/40) = \mathbf{48.75}$ K.
- $A = Q/(U \Delta T_{lm}) = 94\,500/(400 \times 48.75) = \mathbf{4.85}$ m^2.

2.3

- Parallel ends: $90 - 20 = 70$ K and $60 - 31.3 = 28.7$ K; $\Delta T_{lm} = 41.3/\ln(70/28.7) = \mathbf{46.32}$ K.
- $A = 94\,500/(400 \times 46.32) = \mathbf{5.10}$ m^2: counterflow is about 5% smaller.

3.1

- $1/U = 1/U_{clean} + R_f = 1/400 + 0.0005 = 0.003$, so $U = \mathbf{333.3}$ W/(m$^2 \cdot$ K).
- $C_{min} = 1.5 \times 2100 = 3150$ W/K (oil); $C_r = 3150/8360 = \mathbf{0.3768}$.
- $NTU = UA/C_{min} = 333.3 \times 4.85/3150 = \mathbf{0.513}$; counterflow formula gives $\varepsilon = \mathbf{0.377}$.
- $Q = \varepsilon C_{min}(T_{h,in} - T_{c,in}) = 0.377 \times 3150 \times 70 = \mathbf{83.0}$ kW.
- $T_{h,out} = 90 - Q/C_{min} = 90 - 83\,000/3150 = \mathbf{63.6}$ °C.
- Both outlets are unknown, so ΔT_{lm} cannot be found without iterating; ε–NTU needs only the inlets.