10.2 Transient conduction

Name: _____ Class: _____ Date: _____ **Total: 17 marks**

KEY WORDS Biot number • lumped capacitance • Fourier number

Objective

Decide with the Biot number whether a cooling body can be treated as lumped, find its time constant and cooling time, and use the one-term solution when it cannot.

1 Key knowledge and worked examples

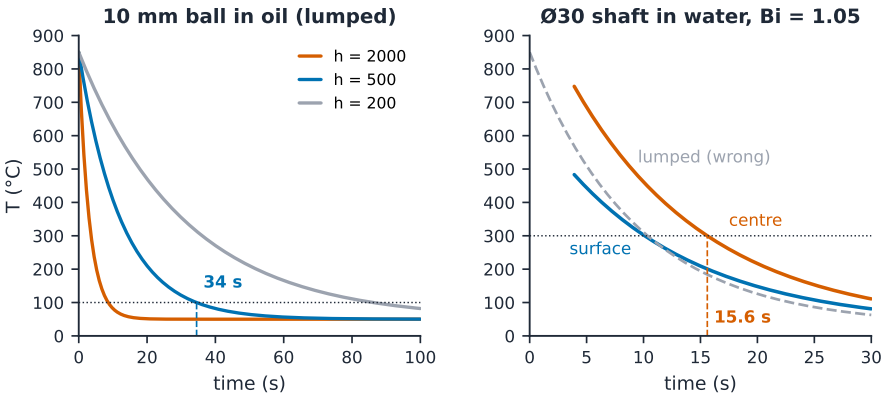
■ **Lumped, or not**

- **Biot number** $Bi = hL_c/k$, $L_c = V/A$ ($r/3$ for a sphere, $r/2$ for a long cylinder).
- **Lumped capacitance** if $Bi < 0.1$: $\theta/\theta_i = e^{-t/\tau}$, $\tau = \rho c L_c/h$.
- Otherwise, one-term solution with the **Fourier number** $Fo = \alpha t/r_o^2$: centre $\theta_0/\theta_i = C_1 e^{-\zeta_1^2 Fo}$ (valid for $Fo > 0.2$).

■ **Worked example. A quenched bearing ball**

Steel ball, 10 mm diameter ($\rho = 7850$, $c = 475$, $k = 43$), oil 50 °C, $h = 500$ W/(m² · K).

- $L_c = r/3 = 1.67$ mm; $Bi = hL_c/k = \mathbf{0.019}$: lumped.
- $\tau = \rho c L_c/h = 7850 \times 475 \times 0.00167/500 = 12.4$ s.
- 850 → 100 °C: $t = \tau \ln(\theta_i/\theta) = 12.4 \ln(800/50) = \mathbf{34}$ s.



2 Practice

Steel: $\rho = 7850$ kg/m³, $c = 475$ J/(kg · K), $k = 43$ W/(m · K).

2.1 A 20 mm steel ball is quenched in oil at 40 °C with $h = 300$ W/(m² · K). Find L_c (two decimals, mm) and Bi (four decimals). Is lumped valid? [3]

2.2 Find the time constant (one decimal, s) and the time to cool from 800 to 100 °C (nearest s). [4]

2.3 Find the ball's temperature 60 s after it enters the oil (nearest °C). [3]

3 Exam-style questions

3.1 A 40 mm steel shaft at 850 °C is quenched in water at 30 °C with $h = 2000$ W/(m^2 · K). Find $Bi = hr_o/k$ and say why lumped fails. For this Bi, $\zeta_1 = 1.221$ and $C_1 = 1.195$. Find the time for the centre to reach 300 °C (one decimal, s), and compare with the lumped estimate (one decimal, s). [7]

Solutions

2.1

- $L_c = r/3 = 10/3 = \mathbf{3.33}$ mm.
- $Bi = hL_c/k = 300 \times 0.00333/43 = \mathbf{0.0233}$, below 0.1: lumped is valid.

2.2

- $\tau = \rho c L_c/h = 7850 \times 475 \times 0.00333/300 = \mathbf{41.4}$ s.
- $t = \tau \ln(\theta_i/\theta) = 41.4 \ln(760/60) = \mathbf{105}$ s.

2.3

- $T = T_\infty + \theta_i e^{-t/\tau} = 40 + 760 e^{-60/41.4} = \mathbf{219}$ °C.

3.1

- $Bi = hr_o/k = 2000 \times 0.020/43 = \mathbf{0.93}$, far above 0.1: the centre lags the surface.
- $\theta_0/\theta_i = 270/820 = 0.329$; $Fo = \ln(C_1/0.329)/\zeta_1^2 = \ln(1.195/0.329)/1.221^2 = 0.865$.
- $\alpha = k/(\rho c) = 1.153 \times 10^{-5}$ m^2/s; $t = Fo\, r_o^2/\alpha = 0.865 \times 0.02^2/(1.153 \times 10^{-5}) = \mathbf{30.0}$ s.
- Lumped: $\tau = \rho c (r/2)/h = 18.6$ s and $t = 18.6 \ln(820/270) = \mathbf{20.7}$ s, a third too short.