10.1 Conduction

Name: _____ Class: _____ Date: _____ **Total: 18 marks**

KEY WORDS Fourier's law • thermal conductivity • thermal resistance • critical radius

Objective

Build thermal-resistance networks for plane walls and insulated pipes, find the heat flow and the temperature at each interface, and use the critical radius.

1 Key knowledge and worked examples

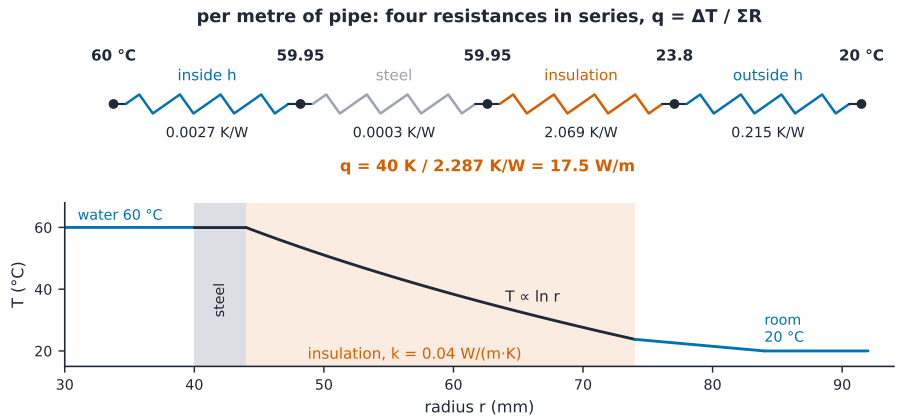
Resistances in series

- **Fourier's law** with **thermal conductivity** k: $q = kA\,\Delta T/L$.
- **Thermal resistance** (K/W): wall $L/(kA)$; convection $1/(hA)$; cylinder $\ln(r_2/r_1)/(2\pi kL)$. In series they add: $q = \Delta T_{total}/\sum R$.
- Each layer's temperature drop $= qR$, so the largest resistance takes the largest drop.
- **Critical radius** $r_c = k/h$: below it, insulation raises the loss.

Worked example. The insulated pipe

Water 60 °C; steel $r_1 = 40$, $r_2 = 44$ mm ($k = 50$); insulation to $r_3 = 74$ mm ($k = 0.04$); $h_i = 1500$, $h_o = 10$ W/(m^2 · K); room 20 °C. Per metre:

- $\sum R = R_i + R_s + R_{ins} + R_o = 0.0027 + 0.0003 + 2.069 + 0.215 = 2.287$ K/W.
- $q = \Delta T/\sum R = 40/2.287 =$ **17.5** W/m, against 110 W/m bare: an 84% saving.



2 Practice

2.1 A concrete wall 0.25 m thick ($k = 1.4$ W/(m · K)) and 10 m^2 in area has faces at 25 K difference. Find its resistance (five decimals, K/W) and the heat flow (nearest W). [3]

2.2 The wall gains 60 mm of insulation ($k = 0.035$), with $h_i = 8$ and $h_o = 20$ W/(m^2 · K); the air temperatures differ by 25 K. Find the total resistance (four decimals, K/W), the heat flow (one decimal, W) and the temperature drop across the insulation (one decimal, K). [5]

2.3 A 5 mm-radius wire is coated with plastic ($k = 0.2$) in air with $h = 10$ W/(m^2 · K). Find the critical radius (mm) and say what a thin coating does to the heat loss. [3]

3 Exam-style questions

3.1 Water at 80 °C flows in a steel pipe ($r_1 = 25$, $r_2 = 28$ mm, $k = 50$) with insulation to $r_3 = 58$ mm ($k = 0.04$); $h_i = 1000$, $h_o = 10$ W/(m^2 · K), room 20 °C. Per metre, find the four resistances (four decimals, K/W), the heat loss (one decimal, W/m) and the outer surface temperature (one decimal, °C). Compare with the bare pipe's loss. [7]

Solutions

2.1

- $R = L/(kA) = 0.25/(1.4 \times 10) = \mathbf{0.01786}$ K/W.
- $q = \Delta T/R = 25/0.01786 = \mathbf{1400}$ W.

2.2

- $R_i = 1/(h_i A) = 1/80 = 0.0125$; $R_{wall} = 0.0179$; $R_{ins} = L/(kA) = 0.06/(0.035 \times 10) = 0.1714$; $R_o = 1/200 = 0.0050$ K/W.
- $\sum R = \mathbf{0.2068}$ K/W; $q = 25/0.2068 = \mathbf{120.9}$ W.
- $\Delta T_{ins} = qR_{ins} = 120.9 \times 0.1714 = \mathbf{20.7}$ K, 83% of the whole drop.

2.3

- $r_c = k/h = 0.2/10 = 0.020$ m $= \mathbf{20}$ mm.
- The wire is below r_c: a coating up to 20 mm radius adds outer area faster than resistance, so it *increases* the loss, useful for cooling cables.

3.1

- $R_i = 1/(h_i 2\pi r_1) = 1/(1000 \times 2\pi \times 0.025) = 0.0064$; $R_s = \ln(r_2/r_1)/(2\pi k) = \ln(28/25)/(2\pi \times 50) = 0.0004$ K/W.
- $R_{ins} = \ln(58/28)/(2\pi \times 0.04) = 2.8976$; $R_o = 1/(10 \times 2\pi \times 0.058) = 0.2744$ K/W.
- $q = \Delta T/\sum R = 60/3.1787 = \mathbf{18.9}$ W/m.
- $T_{surface} = 20 + qR_o = 20 + 18.9 \times 0.2744 = \mathbf{25.2}$ °C.
- Bare: $q = 60/(0.0064 + 0.0004 + 1/(10 \times 2\pi \times 0.028)) = 104.3$ W/m, so insulation saves about 82%.