UNIVERSITY MECHANICAL ENGINEERING • EXERCISE SHEET

1.7 Friction

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS friction • impending slip • static • kinetic

Objective

Decide whether friction holds, and with how much force it actually acts; solve impending-slip problems with correct direction reasoning; and compare tipping against slipping for a pulled or pushed block.

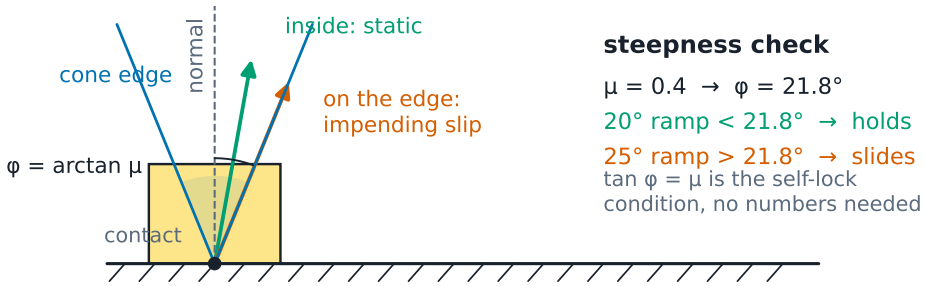
1 Key knowledge and worked examples

■ Friction takes what it needs

Before sliding, dry **friction** adjusts itself to maintain equilibrium, up to $F \leq \mu_s N$ – and while sliding, $F = \mu_k N$ opposes the motion. The inequality is the whole subtlety: the acting friction is whatever equilibrium demands, never automatically μN.

Worked example. A 50 kg block on a 20° incline ($\mu_s = 0.4$). Down-slope weight component: `490.5 sin 20° = 167.8 N`. Available friction: `0.4 × 490.5 cos 20° = 184.3 N`. Since 167.8 < 184.3 the block stays – and the friction actually acting is 167.8 N, not 184.3 N.

The reaction tilts at most arctan(μ) from the normal



■ Tipping versus slipping

A block pulled horizontally at height `h` leaves the ground two ways:

- **Slips** when $P = \mu_s W$.
- **Tips** about its leading edge when $P \cdot h = W \cdot b/2$ (`b` = width along the pull).

The smaller threshold happens first. A tall, narrow, lightly-pulled crate tips; a low, wide, strongly-pulled one slides.

Worked example. $W = 400N$, $b = 0.8m$, $\mu_s = 0.4$, pull at $h = 1.2m$. Slip: $P = 0.4 \times 400 = 160N$. Tip: $P = 400 \times 0.4/1.2 = 133.3N$. It tips first – the pull is too high.

■ Direction before magnitude

At **impending slip** , friction opposes the *would-be* motion. A block being pushed up an incline feels friction down the slope; a ladder foot tending to slide outward feels friction pointing back under the ladder. State the would-be motion in words first – the sign convention then writes itself.

2 Practice

2.1 A 40 kg block rests on a level floor with $\mu_s = 0.35$. A horizontal 100 N push is applied. Will the block move? Give the friction force actually acting. [3]

2.2 A crate weighs 400 N, is 0.8 m wide and is pulled horizontally at a height of 1.2 m. With $\mu_s = 0.4$, find both thresholds and state which occurs first. [5]

2.3 A 200 N block is pushed up a 15° incline (parallel to the slope) with $\mu_s = 0.30$. Find the normal force, and the push needed to hold it in impending motion up the slope. [3]

3 Exam-style questions

3.1 A uniform 600 N ladder leans against a smooth vertical wall. Its foot rests on a floor with $\mu_s = 0.35$. The ladder's own weight is the only load.

Express the wall force in terms of the lean angle θ by moments about the foot, then find the smallest θ at which the ladder does not slip. Verify that at $\theta = 60°$ the available friction exceeds what is needed, and state the acting friction at 60°. [6]

3.2 Block A (20 kg) sits on block B (30 kg), which sits on the floor. The coefficient of static friction is 0.30 between A and B, and 0.25 between B and the floor. A horizontal force P is applied to A.

Find the maximum friction available at each interface, determine whether A slips on B or the pair slides together as P grows, and give the value of P at which motion first occurs. Verify by checking the force transmitted to B against its floor friction. [6]

Solutions

2.1

- Available: $0.35 \times 40 \times 9.81 = 137.3N$.
- Needed to hold 100 N – the block stays.
- Acting friction = 100 N (equal and opposite to the push, not μN).

2.2

- Slip threshold: $P = \mu W = 0.4 \times 400 = 160N$.
- Tip threshold: moments about the leading edge, $P \times 1.2 = 400 \times 0.4 \rightarrow P = 133.3N$.
- $133.3 < 160$: the crate tips first.

2.3

- Normal: $N = 200cos15° = 193.2N$.
- Forces along the slope: push P balances `200 sin 15° = 51.8 N` (down) plus friction $0.30 \times 193.2 = 58.0N$ (down, opposing the would-be upward motion).
- $P = 51.8 + 58.0 = 109.8N$.

3.1

- Smooth wall: only a normal force N_w at the top. Moments about the foot: $N_w \times Lsin\theta = 600 \times (L/2)cos\theta \rightarrow N_w = 300/tan\theta$.
- Horizontal equilibrium: needed floor friction $F = N_w = 300/tan\theta$; available $\mu N_{floor} = 0.35 \times 600 = 210N$.
- Safe when $300/tan\theta \leq 210 \rightarrow tan\theta \geq 1.428 \rightarrow \theta \geq 55.0°$.
- At $\theta = 60°$: $N_w = 300/1.732 = 173.2N < 210N$ ✓; the acting friction is exactly the needed 173.2 N.

3.2

- A–B interface: only A presses – $0.30 \times 20 \times 9.81 = 58.9N$.
- B–floor: both weights press – $0.25 \times 50 \times 9.81 = 122.6N$.
- Suppose no slip at A: the pair needs `P > 122.6 N` to slide.
- But A–B can transmit at most 58.9 N, and $58.9 < 122.6$ – the pair can never be dragged before A slips.
- Motion first occurs at $P = 58.9N$: A slides on B.
- Check: the force reaching B is at most 58.9 N < 122.6 N, so B stays put. ✓