1.5 Rigid-body equilibrium and supports

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS roller • pin • fixed support • statically determinate • two-force member

Objective

Write the three rigid-body equilibrium equations, draw each support's reaction vocabulary without hesitation, and solve complete beams including overhangs – checking every answer through an equation that was not used to produce it.

1 Key knowledge and worked examples

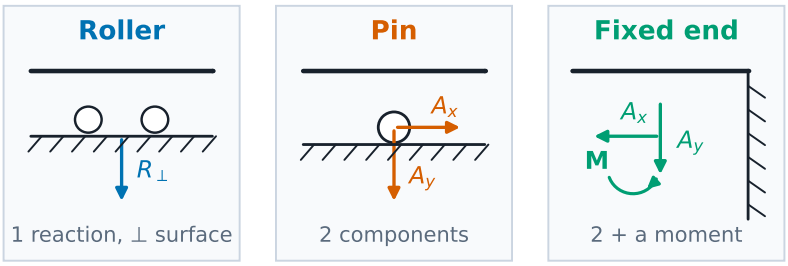
■ Three equations, three kinds of support

A rigid body in 2D gives three equations: $\sum F_x = 0$, $\sum F_y = 0$, $\sum M = 0$.

- **Roller** : one reaction, perpendicular to its surface.
- **Pin** : two components.
- **Fixed support** : two components **and** a moment.

The three equations solve exactly three unknowns – which is why the pin-plus-roller beam is the workhorse of this course: **statically determinate** with nothing spare.

One, two, or two-plus-a-moment: supports priced by what they prevent



■ **Worked example: the support beam**

A 6 m beam, pin at A (left), roller at B (right), 12 kN point load 2 m from A.

- Moments about A: $N_B \times 6 = 12 \times 2 \rightarrow N_B = 4kN$.
- Vertical: $A_y = 12 - 4 = 8kN$. Horizontal: $A_x = 0$ (no horizontal loads).
- Check, moments about B: $A_y \times 6 = 48 = 12 \times 4$. ✓

The check equation was not used to find A_y – that is what makes it a check.

■ **Two-force members do the diagnosis**

A **two-force member** – loaded only at its two ends – carries equal, opposite forces along the line joining the ends. Spotting one removes an unknown direction before any algebra: struts, ties and links are all two-force members by construction.

2 Practice

2.1 Name the number of reaction components supplied by each support and the one extra quantity a fixed support adds: roller on a horizontal floor, pin, fixed support. [3]

2.2 A 5 m beam has a pin at A (left end) and a roller at B (right end), with an 18 kN point load 2 m from A. Find both vertical reactions, taking moments about A first, and verify by moments about B. [4]

2.3 A strut in a frame is pinned at both ends and carries no load along its length. State the direction of the force it exerts on each neighbour, and why. [3]

3 Exam-style questions

3.1 A 7 m beam has a pin at A (left end) and a roller at B, 5 m from A. A 14 kN point load acts at the free right end (7 m from A) – the beam overhangs B by 2 m.

Find the reactions at A and B by moments about A and the vertical equation, and interpret the sign of the reaction at A physically (what holds the beam down?). Verify by moments about B. [6]

3.2 A uniform 5 m ladder of weight 400 N leans at 60° to the horizontal against a smooth vertical wall, with its foot on a rough floor.

Draw (describe) the ladder's FBD (normal force at the wall, normal and friction at the floor, weight at the midpoint), find the wall's normal force by moments about the foot, then find the floor's friction and normal forces. State what would happen if the floor were smooth. [7]

Solutions

2.1

- Roller: one reaction (vertical, for a horizontal floor).
- Pin: two components.
- Fixed support: two components plus a reaction moment – the moment is the extra quantity.

2.2

- Moments about A: $N_B \times 5 = 18 \times 2 \rightarrow N_B = 7.2kN$.
- Vertical: $A_y = 18 - 7.2 = 10.8kN$ (and $A_x = 0$).
- Check, moments about B: $A_y \times 5 = 54 = 18 \times 3$. ✓

2.3

- Along the line joining its two end pins.
- Equilibrium of the strut itself: two forces can only balance if equal, opposite and collinear.
- So each neighbour feels a force along that same line – one unknown instead of two components.

3.1

- FBD: A_x, A_y at A; N_B at 5 m; 14 kN down at 7 m.
- Moments about A: $N_B \times 5 = 14 \times 7 \rightarrow N_B = 19.6kN$ upward.
- Vertical: $A_y = 14 - 19.6 = -5.6kN$ – negative, so 5.6 kN downward.
- Physical reading: the overhang tends to lift the left end about B; the pin must hold the beam down – the pin can pull because it is a pin (a roller could not).
- Check, moments about B: $A_y \times 5 + 14 \times 2 = -28 + 28 = 0$. ✓

3.2

- FBD: N_{wall} horizontal at the top; N_{floor} vertical and friction `F` horizontal at the foot; 400 N down at 2.5 m along the ladder.
- Geometry: ladder top height `5 sin 60° = 4.33 m`; weight's horizontal offset from the foot `2.5 cos 60° = 1.25 m`.
- Moments about the foot: $N_{wall} \times 4.33 = 400 \times 1.25 \rightarrow N_{wall} = 115.5N$.
- Horizontal: $F = N_{wall} = 115.5N$. Vertical: $N_{floor} = 400N$.
- Smooth floor (F = 0): no horizontal balance is possible – the ladder slides out. Friction at the floor is what a ladder stands on.