

1.4 Moments and couples

Name: _____ Class: _____ Date: _____

Total: 22 marks

KEY WORDS moment • moment arm • couple • line of action

Objective

Compute moments about any point with the correct sign, treat couples as point-free turning effects, and use moment equilibrium about deliberately chosen points to eliminate unknowns. Verify through an unused point.

1 Key knowledge and worked examples

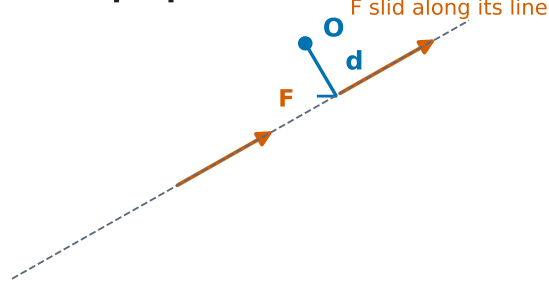
■ Moment = force $\times$ perpendicular distance

The **moment** of a force about a point O is $M = F \cdot d$, with **d** the **moment arm** – the perpendicular distance from O to the force's **line of action**. Counterclockwise is the usual positive sense. When the geometry is not perpendicular, resolve: only the perpendicular component turns the body.

$$\Sigma M_O = 0$$

about any point you like – and choosing O on an unknown's line of action deletes that unknown from the equation.

The arm is the perpendicular from O to the line of action



M about O = $F \cdot d$ for both positions — the perpendicular never changed

■ Worked example: two ways to the same moment

A 300 N force acts at 40° to a beam, applied 1.2 m from support O.

- Direct: $M = 300 \sin 40^\circ \times 1.2 = 231.4 \text{ N} \cdot \text{m}$.
- Resolve first: the perpendicular component $300 \sin 40^\circ = 192.8 \text{ N}$ at 1.2 m – the same $231.4 \text{ N} \cdot \text{m}$, while the 229.8 N along-beam component passes through O and contributes nothing.

The check is free: compute it both ways, once.

■ A couple is the same about every point

A **couple** is two equal, opposite, parallel forces. Its moment is $M = F \times d$ (d between the lines of action) and it is **the same about every point** – a pure turn with no net force. A single force's moment, by contrast, depends on the reference point: 200 N at a 0.5 m arm is $100 \text{ N} \cdot \text{m}$ about that point and something else about any other.

Worked example. Two 150 N forces 0.8 m apart: $M = 120 \text{ N} \cdot \text{m}$ about every point in the plane. Move your moment centre anywhere – the answer does not move.

2 Practice

2.1 A 250 N force acts with a 0.4 m perpendicular moment arm about O, turning counterclockwise. Compute the moment about O with its sign, and the moment about a point on the force's line of action. [3]

2.2 A 300 N force at 40° to a beam is applied 1.2 m from O. Compute the moment about O two ways (perpendicular component first, then directly) and confirm they agree. [4]

2.3 Two 90 N forces form a couple with lines of action 0.5 m apart. Give the couple's moment, then its moment about a point 3 m off centre – and explain why the two answers match. [3]

3 Exam-style questions

3.1 An 8 m beam rests on a pin at A (left) and a roller at B (right). A 20 kN point load acts 3 m from A.

Draw (describe) the FBD, take moments about A to find the reaction at B, take moments about B to find the reaction at A, and confirm with the vertical force equation. State why neither moment equation contains the other support's unknown when you choose the moment centre well. [6]

3.2 A column centre line carries a vertical 500 N force that is applied 0.3 m to the column's right, at the top.

Replace the eccentric force by an equivalent force-couple system on the centre line (magnitude, direction, couple, sense). Verify the equivalence by computing the moment about the column base both ways, and state one practical consequence of an eccentric load on a column. [6]

Solutions

2.1

- $M = 250 \times 0.4 = 100N \cdot m$, positive (counterclockwise).
- About any point on the line of action the moment arm is zero: $M = 0$.

2.2

- Perpendicular component: $300 \sin 40^\circ = 192.8 \text{ N}$.
- Moment: $192.8 \times 1.2 = 231.4N \cdot m$ (counterclockwise).
- Direct: $300 \times 1.2 \times \sin 40^\circ = 231.4N \cdot m$.
- The along-beam component $300 \cos 40^\circ = 229.8 \text{ N}$ has zero arm about O and contributes nothing.

2.3

- Couple: $M = 90 \times 0.5 = 45N \cdot m$.
- About the off-centre point: still $45N \cdot m$.
- Each force's moment changes when the centre moves, but by cancelling amounts – the net force is zero, so the couple's moment is point-free.

3.1

- FBD: A_x, A_y at A; N_B at B; 20 kN down at 3 m from A.
- Moments about A: $N_B \times 8 = 20 \times 3 \rightarrow N_B = 7.5kN$.
- Moments about B: $A_y \times 8 = 20 \times 5 \rightarrow A_y = 12.5kN$.
- Vertical check: $12.5 + 7.5 = 20 \text{ kN}$. ✓
- Each moment centre sits on the other support's line of action, so its unknown has zero arm and drops out of that equation – choosing the centre is choosing which unknown to eliminate.

3.2

- Equivalent system: 500 N down on the centre line, plus a couple $M = 500 \times 0.3 = 150N \cdot m$ clockwise.
- About the base (any point on the centre line): original – 500 N at 0.3 m arm gives $150 \text{ N} \cdot \text{m}$ clockwise; equivalent – the force has zero arm and the couple contributes its $150 \text{ N} \cdot \text{m}$ wherever you stand.
- Practical consequence: the column now carries bending in addition to compression – eccentric loads are why columns are sized for combined stress, not just crushing.