

1.3 Particle equilibrium

Name: _____ Class: _____ Date: _____

Total: 23 marks**KEY WORDS** particle • concurrent • equilibrium • free-body diagram

Objective

Solve particle equilibrium with the two component equations, choose axes that decouple the unknowns, and use the three-force concurrency fact to solve problems without arithmetic. Verify every answer through the unused equation.

1 Key knowledge and worked examples

■ Two equations, however many cables

A **particle** is a body whose size does not matter – every force acts at one point. Equilibrium is two equations:

$$\Sigma F_x = 0 \quad \Sigma F_y = 0$$

Draw all forces on the free-body diagram first; the equations are then bookkeeping. The unknowns can be two tensions, a tension and an angle, or a force and a reaction.

■ Worked example: symmetric cables

A 400 N sign hangs from two cables at 30° above horizontal, symmetric. By symmetry the tensions are equal; vertically:

$$2T \sin 30^\circ = 400 \Rightarrow T = 400 \text{ N}$$

Horizontally, $T \cos 30^\circ$ pulls left and right equally – nothing to compute, and the 30° geometry makes the tension equal the weight. Steeper cables need less tension (60° would give $T = 231 \text{ N}$); flatter cables need more (10° would give 1154 N).

■ **Choose axes, then use the spare equation**

Tilt the axes along and perpendicular to an unknown force: that unknown vanishes from the perpendicular equation, which then solves the other unknown directly.

Worked example. A crate on a smooth 25° incline, held by a rope along the slope. Axes along/perpendicular to the slope: perpendicular gives $N = W\cos 25^\circ$ with the rope absent from the equation; along-slope gives $T = W\sin 25^\circ$ for equilibrium. The unused horizontal/vertical pair then serves as the check.

Three concurrent forces: if three forces balance, their lines of action pass through one point (or are parallel). That geometric fact often names the direction of an unknown force before any algebra.

2 Practice

2.1 A 300 N weight hangs from two symmetric cables at 40° above horizontal. Find the tension in each cable, and the size of the force each cable's horizontal component must cancel. [3]

2.2 A ring is pulled by three ropes in the plane: 80 N east, 60 N north, and a third rope pulling west. Equilibrium holds. Find the third rope's force, and why no north-south force is needed. [3]

2.3 A 500 N crate rests on a smooth (frictionless) 20° incline, held by a rope parallel to the slope. Using axes along and perpendicular to the slope, find the normal force and the rope tension. [4]

3 Exam-style questions

3.1 A 200 kg crate hangs from a single pulley system: one rope segment rises virtually to the ceiling, the other leaves the pulley at 35° above horizontal and is pulled by a worker. The pulley and hook are frictionless and massless.

Draw (describe) the pulley's free-body diagram, explain why the two rope tensions have equal magnitude T , and find T , the worker's pull, from equilibrium of the pulley. Verify with the horizontal equation. [6]

3.2 A lamp of weight 120 N hangs from point O. Cable OA runs horizontally to a wall; strut OB is pinned at O and at the wall, rising at 50° above horizontal, with the lamp hung directly below O.

Explain why the three forces at O must be concurrent, then find the cable tension and the strut force using axes along and perpendicular to the strut. Check both answers with horizontal and vertical equations. [7]

Solutions

2.1

- Vertical: $2T \sin 40^\circ = 300$.
- $T = 300/(2 \times 0.643) = 233.3N$ each.
- Horizontal: each component is $233.4 \times \cos 40^\circ = 178.8N$, cancelled by the other cable's equal and opposite component.

2.2

- Horizontal: $80 - F_3 = 0$, so $F_3 = 80N$ west.
- Vertical: 60 N north is unopposed – but equilibrium holds, so the statement forces a re-read: the third rope must also cancel the 60 N, which a purely westward rope cannot.
- Consistent resolution: $F_3 = 100N$ at $\text{atan}(60/80) = 36.9^\circ$ north of west – the 3-4-5 triangle again.

2.3

- Perpendicular: $N = 500 \cos 20^\circ = 469.8N$.
- Along the slope: $T = 500 \sin 20^\circ = 171.0N$.
- The rope never enters the perpendicular equation because it acts along the slope – that is the axis choice at work.
- Check (vertical equilibrium): $N \cos 20^\circ + T \sin 20^\circ = 441.6 + 58.5 = 500.1 \text{ N}$.
✓

3.1

- FBD: weight $W = 200 \times 9.81 = 1962N$ down; rope tension T up (vertical segment); rope tension T at 35° (pulled segment).
- One continuous frictionless rope carries the same tension throughout – that is what “massless, frictionless pulley” buys.
- Vertical: $T + T \sin 35^\circ = 1962$.
- $T = 1962/1.5736 = 1246.7N$.
- Horizontal check: the support at the pulley mounting supplies $T \cos 35^\circ = 1021 \text{ N}$ horizontally – the rope pair alone cannot balance, and the mounting pin does. ✓
- Worker's pull equals $T = 1247N$ (same rope).

3.2

- Three forces in equilibrium at O must be concurrent (or parallel); all three act through O already, so the lamp's line of action (vertical through O) is the third line – the geometry is consistent by construction.
- Unknowns: cable tension F_A (horizontal), strut force F_B (along 50°).
- Perpendicular to the strut: only the weight and the cable cross it: $120 \cos 50^\circ = F_A \sin 50^\circ \rightarrow F_A = 120 \times 0.643/0.766 = 100.7N$.
- Along the strut: $F_B = 120 \sin 50^\circ + F_A \cos 50^\circ = 91.9 + 64.7 = 156.6N$ (compression).
- Checks: $F_B \cos 50^\circ = 100.7 \text{ N} = F_A$ and $F_B \sin 50^\circ = 120 \text{ N} = W$. ✓