

1.2 Free-body diagrams and support reactions

Name: _____ Class: _____ Date: _____

Total: 23 marks

KEY WORDS free-body diagram

- pin
- fixed support
- load
- reaction
- statically determinate
- roller
- two-force member

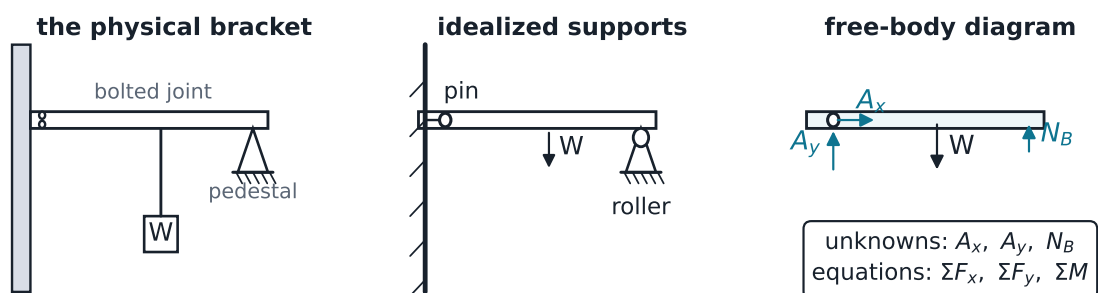
Objective

Construct complete free-body diagrams: isolate one body, draw every external force and support reaction, and audit the diagram against the equilibrium equations before any algebra. Classify what each support transmits.

1 Key knowledge and worked examples

■ Isolate, then draw everything

A **free-body diagram** is a deliberate cut: one body kept, everything else removed, and every force the removed world exerted drawn in its place. Three families belong on it – applied **loads** (weight through the **centre of gravity**, cables, contacts), **reactions** from supports, and forces from members you cut. Nothing internal to the body appears.



■ Each support speaks a fixed language

- **Roller** : one reaction, perpendicular to the surface it bears on.
- **Pin** : two components – draw both, let the signs sort it out.
- **Fixed support** : two components **and** a reaction moment.

Count unknowns, then count equations (two per particle, three per rigid body in 2D). Equal counts make the body **statically determinate** – solvable by equilibrium alone.

■ Worked example: audit before algebra

A 6 m beam has a pin at A (left end), a roller at B (right end) and carries a 12 kN load 2 m from A. The FBD has three unknowns – A_x , A_y , N_B – and three equations: determinate. Moments about A: $N_B \times 6 = 12 \times 2$ gives $N_B = 4kN$; vertically $A_y = 8kN$; horizontally $A_x = 0$ (no horizontal loads).

Audit the diagram itself: no horizontal load exists, so if your FBD invented a horizontal reaction at the roller, the diagram – not the algebra – is already wrong.

2 Practice

2.1 A cable-pulley system holds a 250 N crate with two vertical cable segments sharing it. Draw (describe) the crate's FBD and give the tension in each segment. [3]

2.2 State the number of unknown reaction components for each support: (a) roller on a horizontal surface, (b) pin, (c) fixed support – and name the extra quantity the fixed support adds. [3]

2.3 A 100 kg block rests on a smooth (frictionless) 15° incline. List the forces on the block's FBD with magnitudes ($g = 9.81\text{m/s}^2$), and state why no friction force appears. [4]

3 Exam-style questions

3.1 A 5 m cantilever beam is built into a wall at A (fixed support) and carries a vertical 8 kN point load at its free end.

Draw (describe) the complete FBD with every reaction component, compute all three reactions, and verify the moment reaction by taking moments about the free end. Then explain why the wall must supply a moment at all. [6]

3.2 A frame consists of a horizontal member AB pinned to a wall at A, supported by a two-force strut CD from the wall (at C, above A) to point D on AB, carrying a 6 kN load at B, the free end. Distances: A to D is 1.5 m, D to B is 1.5 m, and CD meets AB at 45° .

Explain what makes CD a **two-force member** and what that implies for the direction of its force on AB. Then compute the strut force and the reactions at A by equilibrium of member AB alone, and check the result by moments about A. [7]

Solutions

2.1

- Forces: weight 250 N down; two cable tensions T_1, T_2 up.
- Equilibrium: $T_1 + T_2 = 250N$.
- Sharing equally (symmetric suspension): $T = 125N$ each.

2.2

- Roller: one unknown (perpendicular to the surface).
- Pin: two unknown components.
- Fixed support: three – two components plus a reaction moment.

2.3

- Weight $W = 100 \times 9.81 = 981N$ straight down at the centre of gravity.
- Normal force $N = 981\cos 15^\circ = 947.6N$ perpendicular to the incline surface.
- Balance: the normal force balances the perpendicular component; the down-slope component $981\sin 15^\circ = 254N$ is unresisted on a smooth surface.
- A smooth surface is defined as frictionless, so no friction force exists; the block would in fact accelerate down the slope.

3.1

- FBD: A_x, A_y , reaction moment M_A at the wall; 8 kN down at the free end.
- Horizontally: $A_x = 0$. Vertically: $A_y = 8kN$ up.
- Moments about A: $M_A = 8 \times 5 = 40kN \cdot m$ (counterclockwise, opposing the load's clockwise moment).
- Check, moments about the free end: $M_A - A_y \times 5 = 40 - 40 = 0$. ✓
- The load's line of action passes 5 m from A; a force pair at A alone ($A_x = A_y$ balanced) could not balance its moment – the wall must also supply a pure couple.

3.2

- CD is loaded only at its two ends (its own weight neglected), so equilibrium of CD itself requires the two forces to be equal, opposite and collinear – along CD.
- Therefore the force on AB at D acts along CD at 45° .
- Member AB FBD: A_x, A_y , force F_{CD} at D at 45° , 6 kN at B.
- Moments about A: $F_{CD}\sin 45^\circ \times 1.5 = 6 \times 3$, so $F_{CD} = 18/1.061 = 16.97 \approx 17.0kN$ compression.
- Horizontally: $A_x = -F_{CD}\cos 45^\circ = -12kN$ (leftward).
- Vertically: $A_y = 6 - F_{CD}\sin 45^\circ = 6 - 12 = -6kN$ (downward).
- Check, ΣM about A already used; ΣM about B: $A_y \times 3 + F_{CD}\sin 45^\circ \times 1.5 = -18 + 18 = 0$. ✓