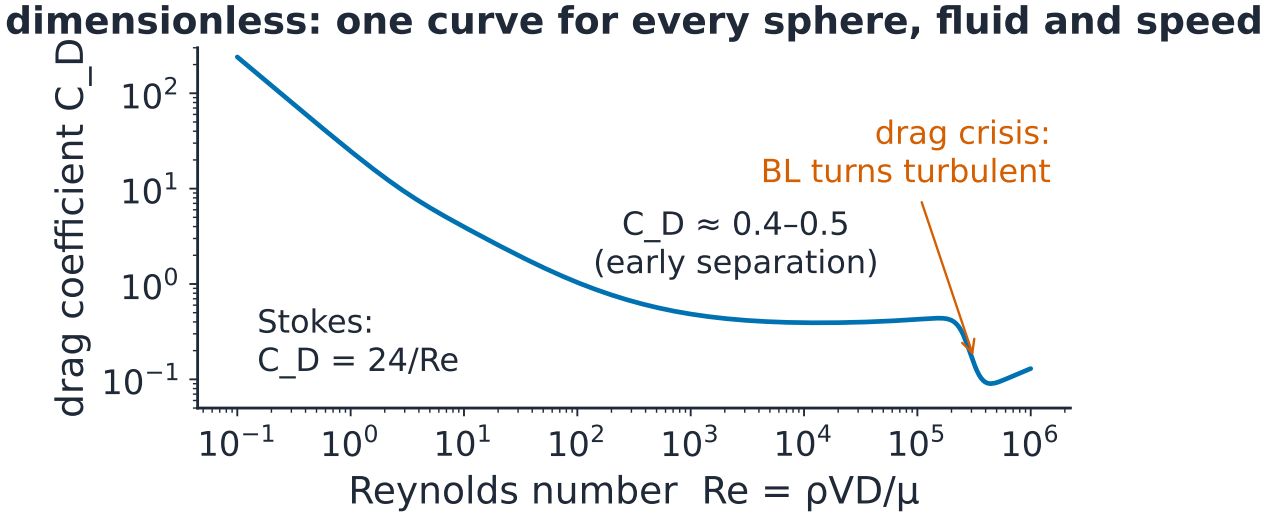
Exercise walk-through

7.4
Dimensional analysis

University Mechanical Engineering

Dimensional analysis: one curve for every sphere



Five variables F, ρ, V, D, μ in three dimensions give two groups: $C_D = F/(\frac{1}{2}\rho V^2 A) = \phi(Re)$, with $Re = VD/\nu$.

The model test in a water tunnel

1. **Prototype** Pod 0.5 m at 20 m/s in air: $Re = 6.60 \times 10^5$.
2. **Model** Half scale in water: $V_m = 20 \times 2 \times (1.004 \times 10^{-6}/1.516 \times 10^{-5}) = \mathbf{2.65}$ m/s.
3. **Scale back** $F_m = 30$ N: $F_p = F_m(\rho_p/\rho_m)(V_p/V_m)^2(L_p/L_m)^2 = \mathbf{8.25}$ N.
4. **Pipe regime** Loop: $Re = 2.39 \times 0.08/1.004 \times 10^{-6} = \mathbf{1.90 \times 10^5}$ - turbulent.

Count the groups

F ρ V D μ — M L T =

$n = 5$ variables $k = 3$ dimensions

$$C_D = \frac{F}{\frac{1}{2}\rho V^2 A}$$

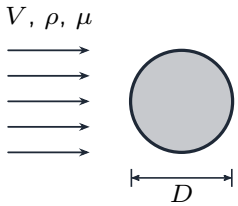
$$Re = \rho V D/\mu$$

measure one curve $C_D(Re)$ and it serves every sphere, fluid and speed

> $C_D = f(Re)$ replaces a function of five variables

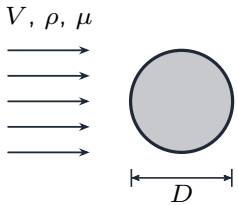
Exercise 2.1

2.1 Write the dimensions of dynamic viscosity μ and of power P. Show that $Re = \dfrac{\rho V D}{\mu}$ has no dimensions. [2]



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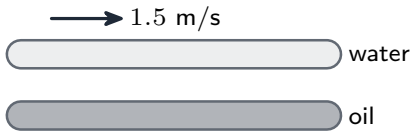


2.1

μ: $ML^{-1}T^{-1}$; P: $ML^{2}T^{-3}$; $\dfrac{ML^{-3} \cdot LT^{-1} \cdot L}{ML^{-1}T^{-1}} = 1$

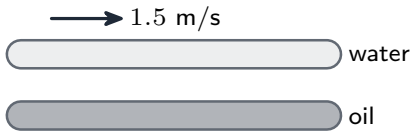
Exercise 2.2

2.2 Water ($\nu = 1.0 \times 10^{-6}$ m^2/s) and an oil ($\nu = 1.0 \times 10^{-4}$ m^2/s) each flow at 1.5 m/s in a 25 mm pipe. Find Re for each and name the regime. [3]



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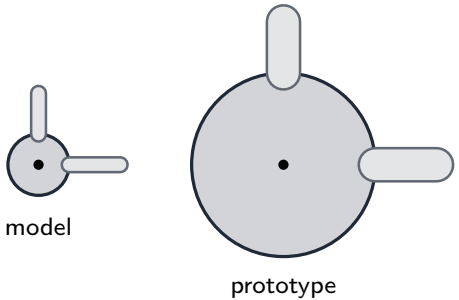


2.2

water: $Re = \dfrac{1.5 \times 0.025}{10^{-6}} = \mathbf{37\,500}$, turbulent; oil: $Re = \mathbf{375}$, laminar; the same pipe and speed, but a viscosity 100 times larger

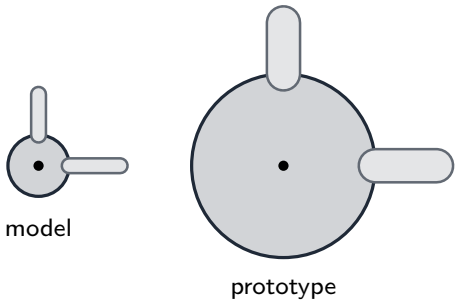
Exercise 2.3

2.3 A model pump of diameter 0.20 m runs at 1500 rpm and takes 3.0 kW. Find the power of a similar pump of diameter 0.60 m at 750 rpm. [3]



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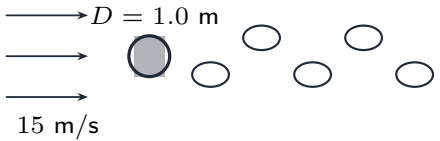


2.3

$\dfrac{P}{\rho N^3 D^5}$ the same; $P_p = 3.0 \times \left(\dfrac{750}{1500}\right)^3 \times 3^5 = \mathbf{91\ kW}$

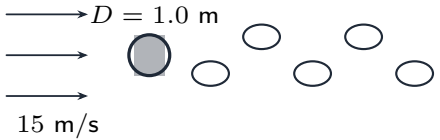
Exercise 2.4

2.4 Wind at 15 m/s blows past a chimney 1.0 m in diameter. Vortices are shed at a Strouhal number $St = \dfrac{fD}{V} = 0.20$. Find the shedding frequency, and say why it matters. [2]



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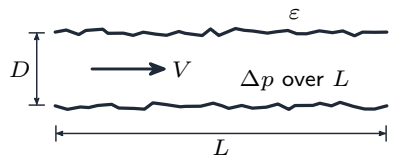
2.4

$f = \dfrac{0.20 \times 15}{1.0} = \mathbf{3.0\ Hz}$; if it is near the chimney's natural frequency, the chimney can resonate

Exercise 3.1

3.1 The pressure drop Δp in a straight pipe depends on ρ, V, D, μ, the length L and the wall roughness ε.

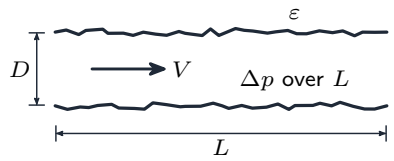
Use the Buckingham Π method to find the dimensionless groups, and write the result as one equation. [5]



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3.1

$n = 7$, $k = 3$: four groups; repeat ρ, V, D

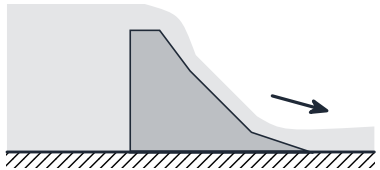
$$\Pi_1 = \frac{\Delta p}{\rho V^2};\ \Pi_2 = \frac{\rho V D}{\mu};\ \Pi_3 = \frac{L}{D},\ \Pi_4 = \frac{\varepsilon}{D}$$

$$\text{so } \frac{\Delta p}{\rho V^2} = \phi\left(Re, \frac{L}{D}, \frac{\varepsilon}{D}\right)$$

Exercise 3.2

3.2 A 1:30 model of a dam spillway is built. The design flood is 450 m^3/s.

Find the model flow rate. A velocity of 1.2 m/s is measured on the model; find the full-size velocity. Find the time scale. [4]

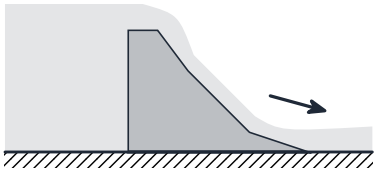


model 1:30

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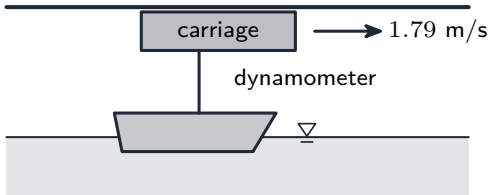
3.2

free surface: Froude similarity; $Q_m = \dfrac{450}{30^{2.5}} = \mathbf{0.091\ m^3/s}$; $V_p = 1.2\sqrt{30} = \mathbf{6.6\ m/s}$; $t_p = \sqrt{30}\,t_m = 5.5\,t_m$

Exercise 3.3 (a)

3.3 A ship 100 m long is to sail at 8.0 m/s in sea water ($\rho = 1025$ kg/m^3, $\nu = 1.19 \times 10^{-6}$ m^2/s). A 1:20 model is towed in fresh water ($\nu = 1.0 \times 10^{-6}$ m^2/s); its wetted area is 3.2 m^2 and its measured resistance is 40 N at the right speed. Use the friction line $C_f = \dfrac{0.075}{(\log_{10} Re - 2)^2}$.

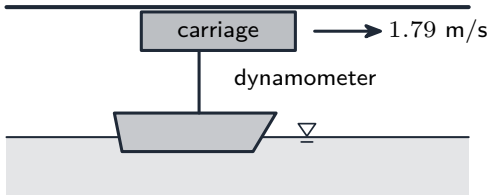
(a) Find the model speed. Find Re for model and ship, and explain why both Fr and Re cannot be matched. [3]



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(a) Find the model speed. Find Re for model and ship, and explain why both Fr and Re cannot be matched. [3]



(a) $V_m = \dfrac{8.0}{\sqrt{20}} = \mathbf{1.79\ m/s}$; $Re_m = 8.9 \times 10^6$, $Re_s = 6.7 \times 10^8$; matching Re as well would need a fluid with ν about 90 times smaller than water, which does not exist

Exercise 3.3 (b)

3.3 A ship 100 m long is to sail at 8.0 m/s in sea water ($\rho = 1025$ kg/m^3, $\nu = 1.19 \times 10^{-6}$ m^2/s). A 1:20 model is towed in fresh water ($\nu = 1.0 \times 10^{-6}$ m^2/s); its wetted area is 3.2 m^2 and its measured resistance is 40 N at the right speed. Use the friction line $C_f = \dfrac{0.075}{(\log_{10} Re - 2)^2}$.

(b) Find C_f for the model and the ship. [2]

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(b) Find C_f for the model and the ship. [2]

(b) $C_{f,m} = \mathbf{0.00306}$, $C_{f,s} = \mathbf{0.00161}$

Exercise 3.3 (c)

3.3 A ship 100 m long is to sail at 8.0 m/s in sea water ($\rho = 1025$ kg/m^3, $\nu = 1.19 \times 10^{-6}$ m^2/s). A 1:20 model is towed in fresh water ($\nu = 1.0 \times 10^{-6}$ m^2/s); its wetted area is 3.2 m^2 and its measured resistance is 40 N at the right speed. Use the friction line $C_f = \dfrac{0.075}{(\log_{10} Re - 2)^2}$.

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(c) model friction $\tfrac{1}{2} \times 1000 \times 1.79^2 \times 3.2 \times 0.00306 = 15.7$ N, residual 24.3 N; ship: residual $24.3 \times 20^3 \times 1.025 = 200$ kN, friction $\tfrac{1}{2} \times 1025 \times 8^2 \times 1280 \times 0.00161 = 67.5$ kN; total **267 kN**, power $267 \times 8.0 =$ **2.1 MW**