Exercise walk-through

4.6
Failure and material selection

University Mechanical Engineering

Bonding sets the property family

metallic
steel, aluminium
ductile, tough, conducts heat and charge

covalent / ionic
alumina, silicon carbide
stiff, hard, brittle, insulating, heat-resistant

polymer chains
nylon, epoxy
light, compliant, viscoelastic, creeps

> Polymer chains give light, viscoelastic solids

Selection in action: one method, two answers

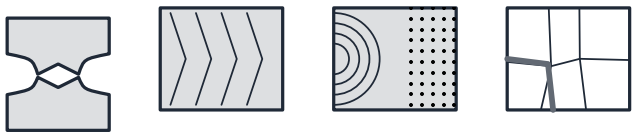
1. **Rank** $\sqrt{E}/\rho$: CFRP 7.37 wins stiffness-per-mass at 2.4× aluminium.
2. **Eliminate** Plain nylon fails the T_g constraint; ceramics fail the survival probability at these loads.
3. **Split** The mount ships quenched-and-tempered steel (620 MPa, 16%); the carry-handle ships CFRP.

The project sheet, reproducible

1. **Structure** BCC 0.6802, FCC 0.7405; Hall-Petch 183.6 $\rightarrow$ 262.3 MPa for 50 $\rightarrow$ 10 µm.
2. **Certificate** Rod 191.0 MPa: 0.477 vs 1.364 mm; true ln 1.2 $=$ 0.1823; **toughness** 92.8 MJ/m^3.
3. **Choice** Routes 280/24, 260/26, 620/16, 950/8; Chvorinov 22.2 vs 50.0 s; Basquin 303.1 at 10^5, 250 at 10^6; Goodman 175.0; index 7.37 / 3.10 / 1.80 / 1.23 - steel mount, CFRP handle.

Exercise 2.1

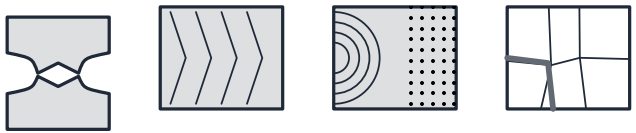
2.1 The figure shows four fracture surfaces, A to D. Name the failure mode of each, and give the evidence. [3]



A *B* *C* *D*

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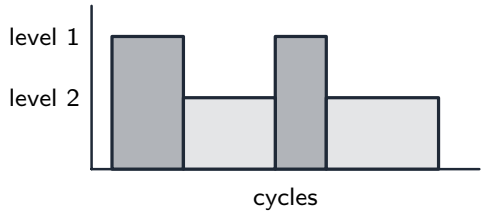
A *B* *C* *D*

2.1

A ductile (neck, cup and cone); B brittle (flat, chevrons point to the origin); C fatigue (beach marks from the origin); D intergranular corrosion (attack along the grain boundaries)

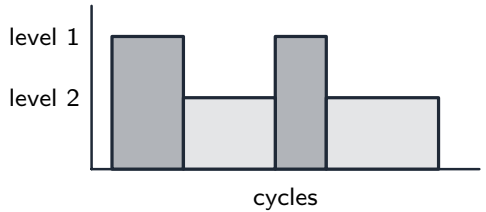
Exercise 2.2

2.2 A part has $N_1 = 1.0 \times 10^5$ cycles of life at stress level 1 and $N_2 = 1.0 \times 10^6$ at level 2. It has already done 4.0×10^4 cycles at level 1 and 3.0×10^5 at level 2. Use Miner's rule to find how many more cycles it can do at level 1. [3]



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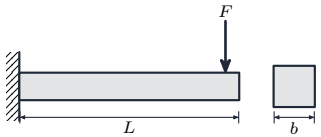


2.2

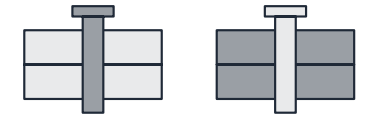
$$\text{damage so far} = \frac{4.0 \times 10^4}{10^5} + \frac{3.0 \times 10^5}{10^6} = 0.70;\ \text{left } 0.30 \times 10^5 = \mathbf{3.0 \times 10^4\ cycles}$$

Exercises 2.3·2.4

2.3 A light stiff beam is needed. Rank aluminium (70 GPa, 2.7 Mg/m^3), steel (200 GPa, 7.85), CFRP (110 GPa, 1.6) and wood (12 GPa, 0.60) by $E^{1/2}/\rho$. [3]

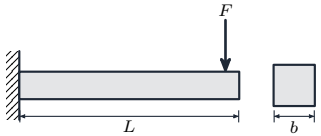


2.4 A steel bolt holds a copper plate, and in another joint a copper bolt holds a steel plate. Both are wet with sea water. Say which joint is worse and why. [2]



Exercises 2.3·2.4

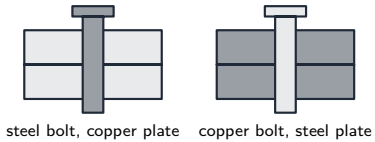
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2.3

$E^{1/2}/\rho$: Al 3.1, steel 1.8, CFRP 6.6, wood 5.8; order: **CFRP > wood > Al > steel**

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2.4

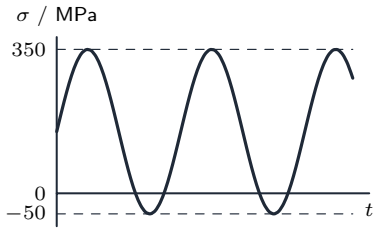
steel is the anode and corrodes; the **steel bolt in copper** is worse: a small anode next to a large cathode is attacked fast, and the joint fails

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Exercise 3.1

3.1 A connecting rod has $S_u = 900$ MPa and a corrected endurance limit $S_e = 380$ MPa. Its stress goes from -50 MPa to $+350$ MPa in every cycle. The design code asks for $n \geq 1.5$.

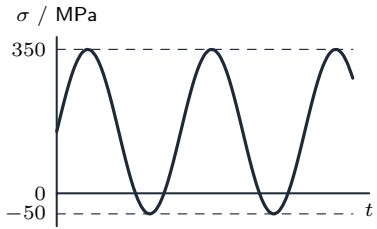
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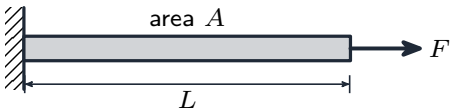
$\sigma_m = 150$ MPa, $\sigma_a = 200$ MPa; $\dfrac{1}{n} = \dfrac{200}{380} + \dfrac{150}{900} = 0.693$, $n = \mathbf{1.44}$

$1.44 < 1.5$: **not acceptable**; need $\dfrac{200}{S_e} = 0.667 - 0.167$, $S_e \geq \mathbf{400\ MPa}$, for example by shot peening

Exercise 3.2

3.2 A tie of length L must carry a force F without yielding, with the least mass. Its cross-section area A is free.

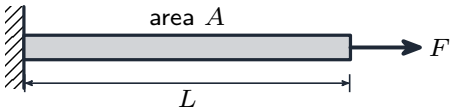
Derive the index to maximise. Then rank steel (350 MPa, 7.85 Mg/m^3), aluminium alloy (300, 2.7), CFRP (600, 1.6) and titanium alloy (900, 4.5). [4]



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3.2

objective $m = AL\rho$; constraint $\dfrac{F}{A} \le \sigma_f$, so $A = \dfrac{F}{\sigma_f}$; $m = FL\dfrac{\rho}{\sigma_f}$: maximise $\sigma_\mathbf{f}/\rho$

steel 45, Al 111, CFRP 375, Ti 200; **CFRP > Ti > Al > steel**

Exercise 3.3 (a)

3.3 A cylindrical air receiver has radius $r = 0.20$ m, wall thickness $t = 10$ mm and pressure $p = 10$ MPa. Steel A has $\sigma_y = 900$ MPa and $K_{IC} = 30$ MPa $\mathrm{m}^{1/2}$; steel B has $\sigma_y = 400$ MPa and $K_{IC} = 80$ MPa $\mathrm{m}^{1/2}$. A vessel is safe by **leak-before-break** if its critical crack size $a_c = \dfrac{1}{\pi}\left(\dfrac{K_{IC}}{\sigma}\right)^2$ is larger than t: a crack then leaks gas, and is found, before it can cause a burst.

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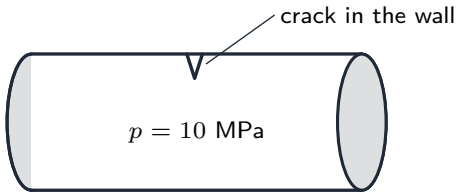
(a) Find the hoop stress $\sigma = \dfrac{pr}{t}$, and the factor of safety on yield for each steel. [2]

(a) $\sigma = \dfrac{10 \times 200}{10} = \mathbf{200\ MPa}$; A: 4.5, B: 2.0

Exercise 3.3 (b)

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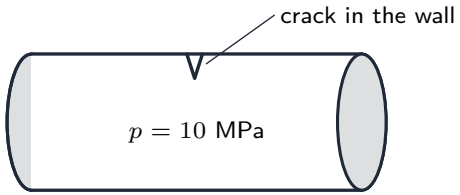
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(b) A: $a_c = \frac{1}{\pi}\left(\frac{30}{200}\right)^2 = \mathbf{7.2\ mm} < 10$ mm; B: $\mathbf{51\ mm} > 10$ mm: only B leaks before it breaks

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Exercise 3.3 (c)

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(c) Choose a steel. Explain why the stronger steel is the wrong choice here. [3]

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(c) Choose a steel. Explain why the stronger steel is the wrong choice here. [3]

(c) choose B; in A a crack bursts the vessel before it is through the wall, with no warning; the strength margin of A does not help, because failure is set by toughness, not by yield