Exercise walk-through

3.4
Impulse and momentum

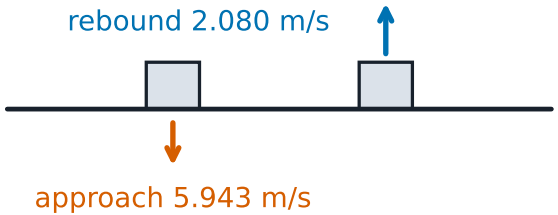
University Mechanical Engineering

Impulse and momentum: the impact file

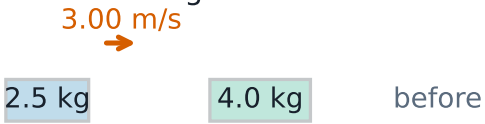
Certified impact: e = 0.35

rebound height 0.2205 m;
87.8% of arrival energy absorbed

J = 20.06 N s
F_avg = 2507 N
(8.0 ms)



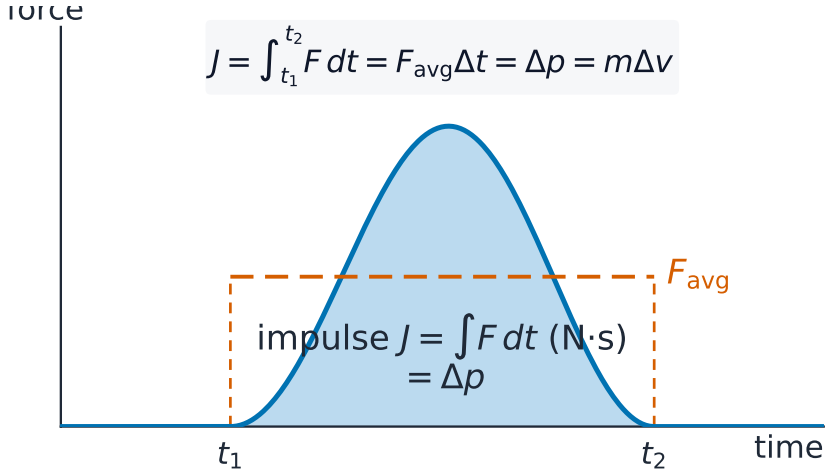
Marshalling test: e = 0.60



momentum 7.500 kept, 4.431 J lost;
fan: 0.120 kg/s × 8.50 m/s
= 1.020 N thrust

Impulse 20.06 N · s brings 2507 N over 8 ms; the carts keep momentum 7.500.

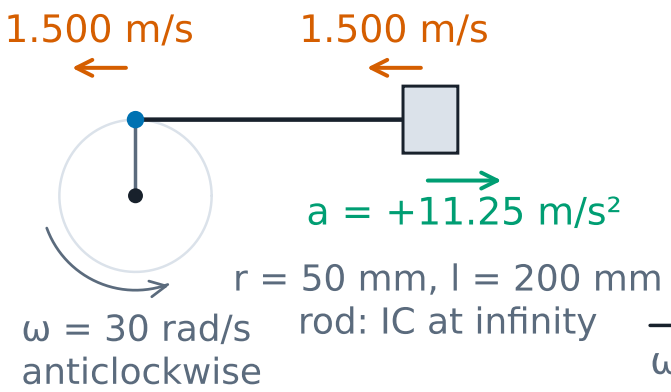
Impulse is the area under F-t



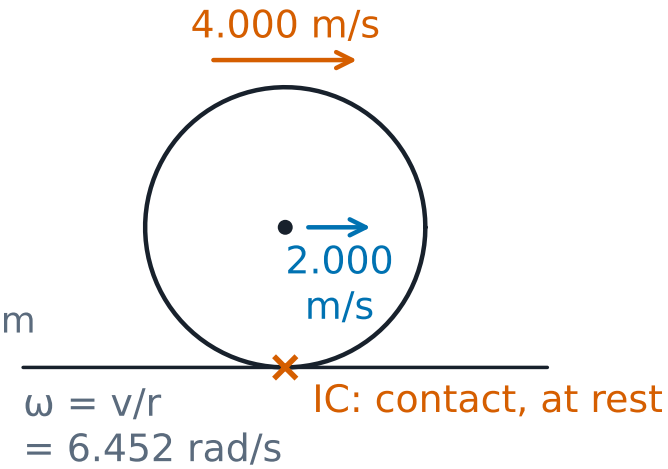
A longer contact time means a smaller peak force

Rigid-body kinematics: constraint first

Slider-crank at 90°



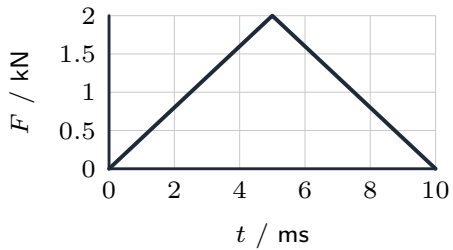
Rolling wheel



At $\theta = 90°$ the slider moves at $\mathbf{-1.500}$ m/s and accelerates at $\mathbf{+11.62}$ m/s^2.

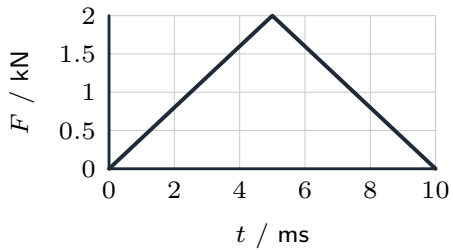
Exercise 2.1

2.1 A 0.50 kg puck at rest is struck by a stick. The force rises steadily to 2.0 kN and falls back to zero; the whole contact lasts 10 ms. Find the impulse and the speed of the puck. [3]



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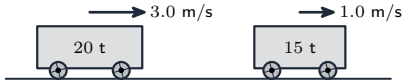


2.1

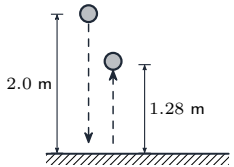
impulse $=$ area $= \frac{1}{2} \times 0.010 \times 2000 = \mathbf{10\ N\ s}$; $v = \dfrac{10}{0.50} = \mathbf{20\ m/s}$

Exercises 2.2·2.3

2.2 A 20 t rail wagon at 3.0 m/s runs into a 15 t wagon moving the same way at 1.0 m/s. They couple together. Find their common speed and the kinetic energy lost. [4]

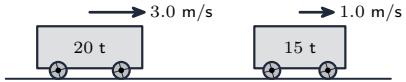


2.3 A ball is dropped from 2.0 m onto a hard floor and bounces to 1.28 m. Find the coefficient of restitution. [2]



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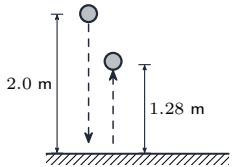


2.2

$20 \times 3.0 + 15 \times 1.0 = 35v$; $v = \mathbf{2.14\ m/s}$

$T_1 = 90 + 7.5 = 97.5$ kJ, $T_2 = \frac{1}{2} \times 35\,000 \times 2.14^2 = 80.4$ kJ; loss = $\mathbf{17.1\ kJ}$

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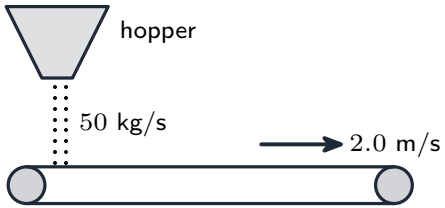


2.3

$$e = \frac{v_{up}}{v_{down}} = \sqrt{\frac{h_2}{h_1}} = \sqrt{0.64} = \mathbf{0.80}$$

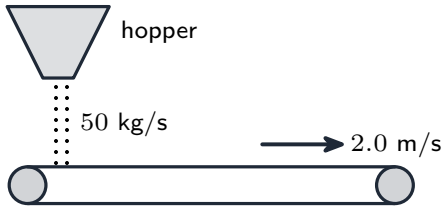
Exercise 2.4

2.4 Sand falls from a hopper onto a conveyor belt at 50 kg/s. The belt moves at 2.0 m/s. Find the extra force needed to keep the belt moving, the extra power, and the rate at which the sand gains kinetic energy. [3]



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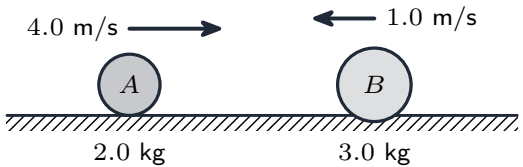
2.4

$F = \dot{m}\,v = 50 \times 2.0 = \mathbf{100\ N}$; $P = Fv = \mathbf{200\ W}$; $\frac{1}{2}\dot{m}v^2 = \mathbf{100\ W}$: half the power is lost as heat while the sand slips on the belt

Exercise 3.1

3.1 Puck A (2.0 kg) moves at 4.0 m/s towards puck B (3.0 kg), which moves at 1.0 m/s the other way. They hit head-on with $e = 0.60$.

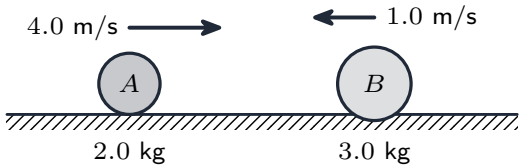
Find both velocities after the impact and the kinetic energy lost. [5]



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Find both velocities after the impact and the kinetic energy lost. [5]



3.1

momentum: $2.0(4.0) + 3.0(-1.0) = 2.0v'_A + 3.0v'_B$, so $2v'_A + 3v'_B = 5$

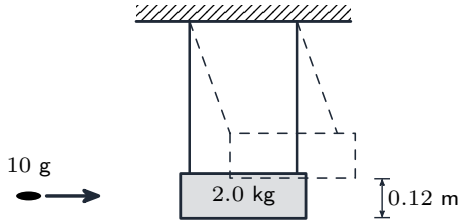
restitution: $v'_B - v'_A = 0.60 \times 5.0 = 3.0$; $v'_A = \mathbf{-0.80\ m/s}$ (back to the left), $v'_B = \mathbf{2.2\ m/s}$

$T_1 = 17.5$ J, $T_2 = 0.64 + 7.26 = 7.9$ J: loss **9.6 J**

Exercise 3.2

3.2 A 10 g bullet is fired into a 2.0 kg wooden block that hangs on light cords. The block, with the bullet inside, swings up through a height of 0.12 m.

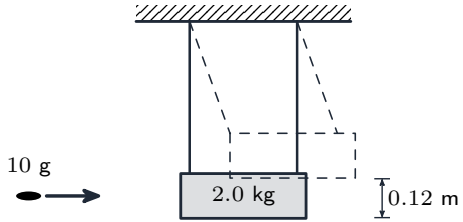
Find the speed of the bullet. Explain why energy cannot be used across the impact, and find the fraction of the bullet's energy that is kept. [4]



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3.2

after the impact: $v = \sqrt{2gh} = 1.53$ m/s; momentum across the impact: $0.010u = 2.010 \times 1.53$, $u = \mathbf{308\ m/s}$

the impact is plastic: much energy goes to heat and to damage in the wood; kept fraction $= \dfrac{m}{m+M} = \mathbf{0.5}\%$

Exercise 3.3 (a)

3.3 A small hydro plant uses a Pelton wheel . A water jet of diameter 50 mm and speed $v = 30$ m/s hits buckets that move at speed u and turn the water back through 165°. Take the whole jet flow $\dot{m} = \rho A v$ as used by the wheel.

(a) Find $\dot{m}$, and the force on the buckets when the wheel is held still. [2]

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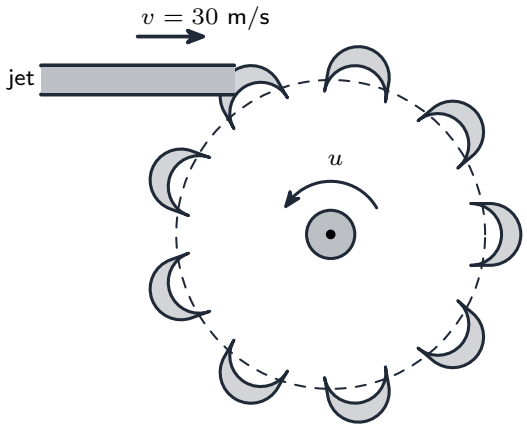
(a) Find $\dot{m}$, and the force on the buckets when the wheel is held still. [2]

(a) $\dot{m} = 1000 \times \pi(0.025)^2 \times 30 = \mathbf{58.9\ kg/s}$; $F = 58.9 \times 30 \times 1.966 = \mathbf{3.47\ kN}$

Exercise 3.3 (b)

3.3 A small hydro plant uses a Pelton wheel . A water jet of diameter 50 mm and speed $v = 30$ m/s hits buckets that move at speed u and turn the water back through 165°. Take the whole jet flow $\dot{m} = \rho A v$ as used by the wheel.

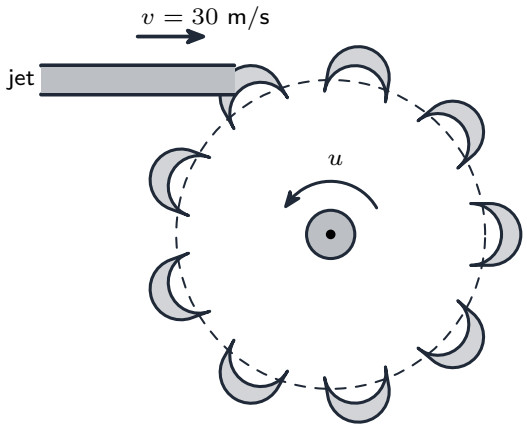
(b) The force is $F = \dot{m}(v - u)(1 + \cos 15°)$. Find the power at $u = 12$ m/s. [3]



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(b) The force is $F = \dot{m}(v - u)(1 + \cos 15°)$. Find the power at $u = 12$ m/s. [3]



(b) $F = 58.9 \times 18 \times 1.966 = 2085$ N; $P = Fu = \mathbf{25.0\ kW}$

Exercise 3.3 (c)

3.3 A small hydro plant uses a Pelton wheel . A water jet of diameter 50 mm and speed $v = 30$ m/s hits buckets that move at speed u and turn the water back through 165°. Take the whole jet flow $\dot{m} = \rho A v$ as used by the wheel.

(c) Show that the power is largest at $u = v/2$, find this power as a percentage of the jet power $\tfrac{1}{2}\dot{m}v^2$, and find the wheel's pitch diameter for 750 rpm. [3]

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(c) $P \propto (v-u)u$, $\dfrac{dP}{du} = v - 2u = 0$; $P_{max} = 26.1$ kW of 26.5 kW: **98**%; $D = \dfrac{2u}{\omega} = \dfrac{30}{78.5} =$ **0.38 m**