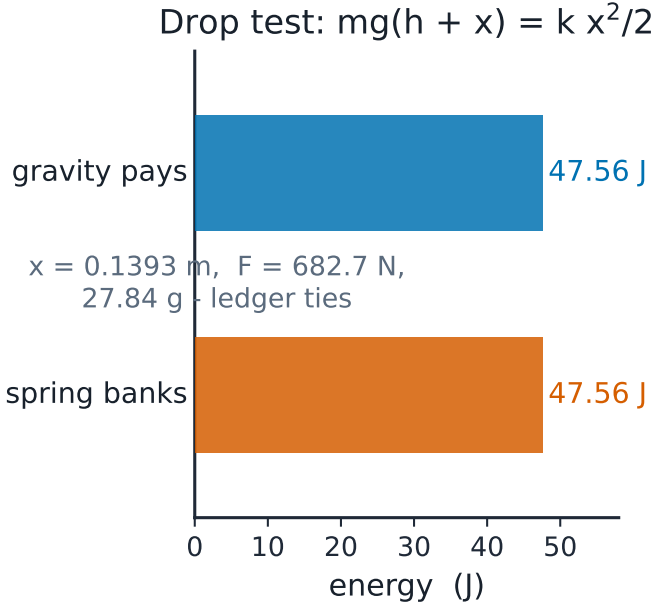
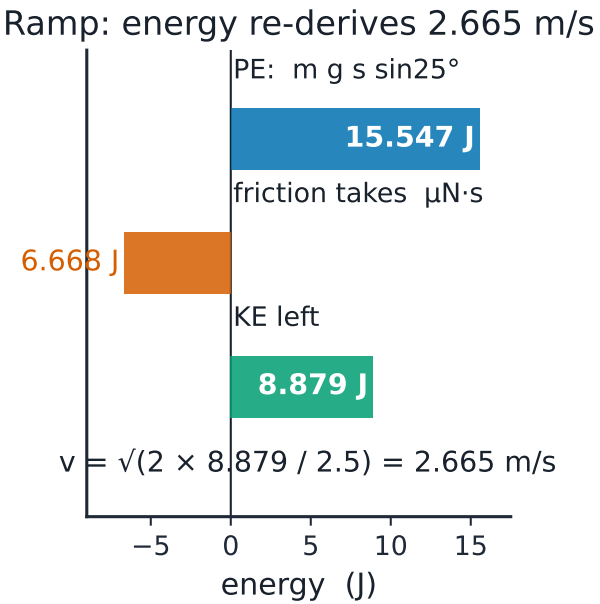
Exercise walk-through

3.1
Particle kinematics

University Mechanical Engineering

Work and energy: the ledger must tie





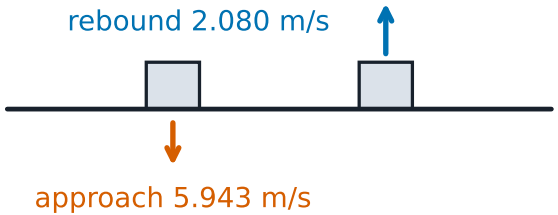
The drop ledger balances at **47.56** J; the ramp re-derives **2.665** m/s.

Impulse and momentum: the impact file

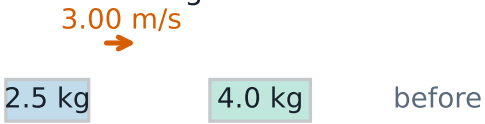
Certified impact: e = 0.35

rebound height 0.2205 m;
87.8% of arrival energy absorbed

J = 20.06 N s
F_avg = 2507 N
(8.0 ms)



Marshalling test: e = 0.60

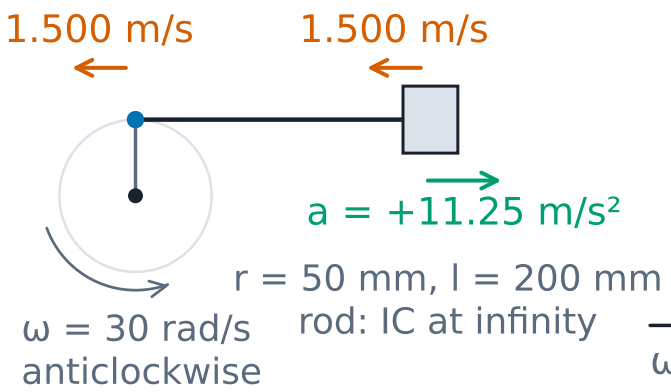


momentum 7.500 kept, 4.431 J lost;
fan: 0.120 kg/s × 8.50 m/s
= 1.020 N thrust

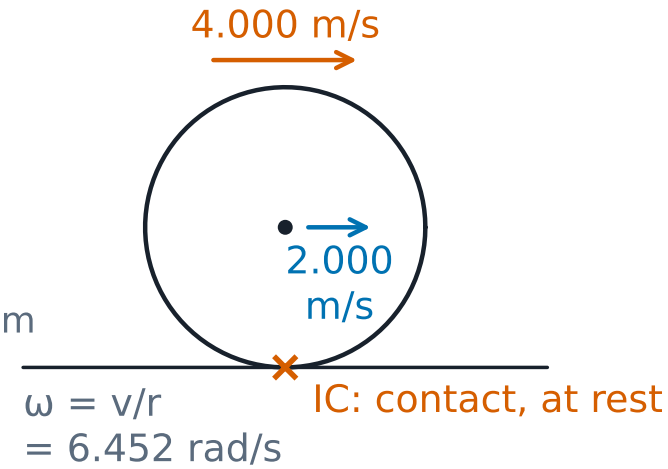
Impulse 20.06 N · s brings 2507 N over 8 ms; the carts keep momentum 7.500.

Rigid-body kinematics: constraint first

Slider-crank at 90°



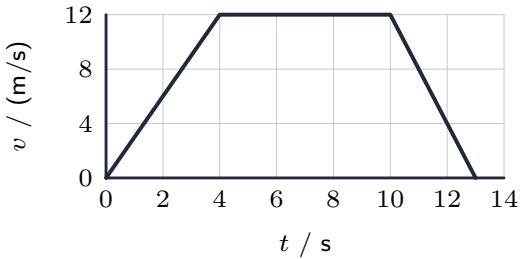
Rolling wheel



At $\theta = 90°$ the slider moves at $\mathbf{-1.500}$ m/s and accelerates at $\mathbf{+11.62}$ m/s^2.

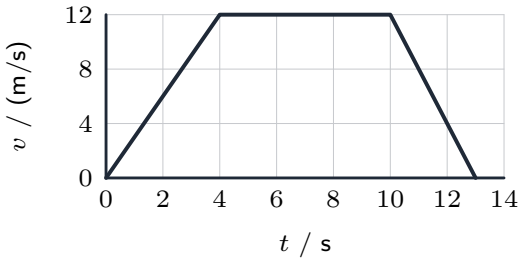
Exercise 2.1

2.1 A lift's speed follows the v–t graph: it speeds up from rest to 12 m/s in 4 s, runs at 12 m/s for 6 s, and stops in 3 s. Find the distance travelled and the largest acceleration. [3]



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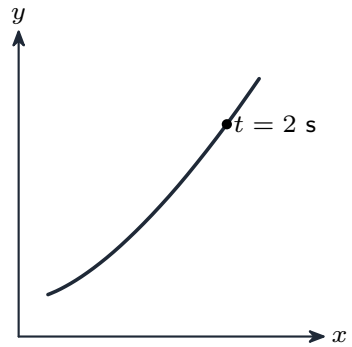


2.1

distance = area = $\frac{1}{2}(4)(12) + 6(12) + \frac{1}{2}(3)(12) = \mathbf{114\ m}$; steepest slope $\dfrac{12}{3} = \mathbf{4.0\ m/s^2}$ (braking)

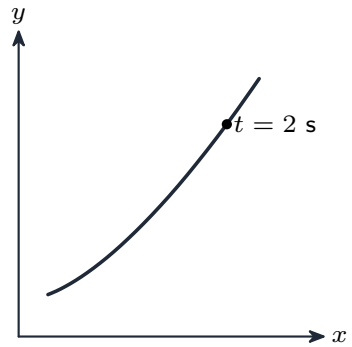
Exercise 2.2

2.2 A drone moves with $\vec{r} = 3t^2\,\hat{\imath} + (2t^3 - t)\,\hat{\jmath}$ (metres, t in seconds). Find its velocity and acceleration at $t = 2$ s, with their sizes. [3]



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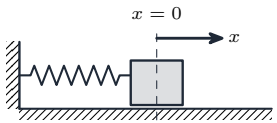
2.2

$\vec{v} = 6t\,\hat{\imath} + (6t^2 - 1)\,\hat{\jmath} = (12,\ 23)$ m/s, $|\vec{v}| = \mathbf{25.9\ m/s}$

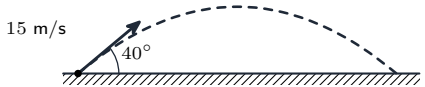
$\vec{a} = 6\,\hat{\imath} + 12t\,\hat{\jmath} = (6,\ 24)$ m/s^2, $|\vec{a}| = \mathbf{24.7\ m/s^2}$

Exercises 2.3·2.4

2.3 A slider on a spring has $a = -25x$ (m/s^2, x in metres). It passes $x = 0$ at 2.0 m/s. Use $a = v\dfrac{dv}{dx}$ to find how far it moves before it stops. [3]

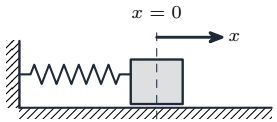


2.4 A ball is kicked from level ground at 15 m/s, 40° above the horizontal. Find its range and its greatest height. [3]



Exercises 2.3·2.4

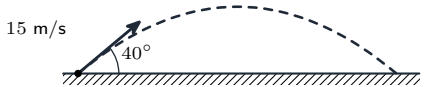
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2.3

$$\int_2^0 v\,dv = \int_0^x -25x\,dx;\ -2 = -12.5x^2;\ x = \mathbf{0.40\ m}$$

2.4 A ball is kicked from level ground at 15 m/s, 40° above the horizontal. Find its range and its greatest height. [3]



2.4

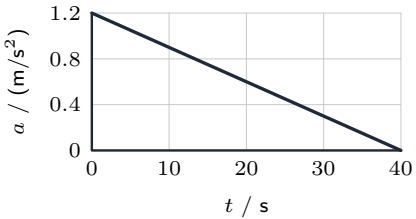
$$\text{range} = \frac{v^2 \sin 2\theta}{g} = \frac{225 \sin 80^\circ}{9.81} = \mathbf{22.6\ m};\ \text{height} = \frac{(15 \sin 40^\circ)^2}{2g} = \mathbf{4.74\ m}$$

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Exercise 3.1

3.1 A train starts from rest. Its acceleration falls steadily: $a = 1.2\left(1 - \dfrac{t}{40}\right)$ m/s^2 for $0 \le t \le 40$ s.

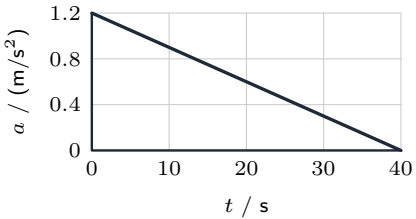
Find its speed at $t = 40$ s and the distance it has travelled. Check the speed from the graph. [4]



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3.1

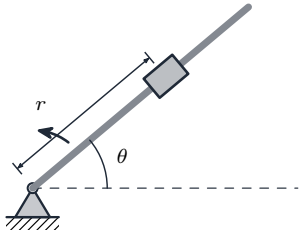
$v = \displaystyle\int_0^t a\,dt = 1.2\left(t - \frac{t^2}{80}\right)$, $v(40) = \mathbf{24\ m/s}$; check: area of the triangle $\frac{1}{2} \times 40 \times 1.2 = 24$

$x = 1.2\left(\dfrac{t^2}{2} - \dfrac{t^3}{240}\right)$, $x(40) = \mathbf{640\ m}$

Exercise 3.2

3.2 A collar slides out along a robot arm while the arm turns. Its position is $r = 0.50 + 0.20t$ m and $\theta = 0.60t$ rad.

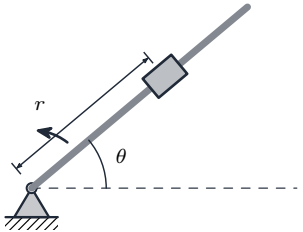
At $t = 2.0$ s, find the radial and transverse parts of the velocity and of the acceleration, and the size of the acceleration. Say which term would be missing if the collar did not slide. [5]



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3.2

$r = 0.90$ m, $\dot{r} = 0.20$ m/s, $\ddot{r} = 0$, $\dot{\theta} = 0.60$ rad/s, $\ddot{\theta} = 0$; $v_r = \mathbf{0.20\ m/s}$, $v_\theta = r\dot{\theta} = \mathbf{0.54\ m/s}$

$a_r = \ddot{r} - r\dot{\theta}^2 = \mathbf{-0.324\ m/s^2}$ (inwards); $a_\theta = 2\dot{r}\dot{\theta} = \mathbf{0.24\ m/s^2}$

$|\vec{a}| = \mathbf{0.403\ m/s^2}$; without sliding the Coriolis term $2\dot{r}\dot{\theta}$ is zero

Exercise 3.3 (a)

3.3 A car enters a curve of radius 60 m at 25 m/s. It brakes steadily, and is at 15 m/s after 80 m along the curve. The tyres can give a total acceleration of at most $0.80g$.

(a) Find the tangential acceleration a_t. [2]

Exercise 3.3 (a)

3.3 A car enters a curve of radius 60 m at 25 m/s. It brakes steadily, and is at 15 m/s after 80 m along the curve. The tyres can give a total acceleration of at most $0.80g$.

(a) Find the tangential acceleration a_t. [2]

(a) $v_2^2 = v_1^2 + 2a_t s$: $a_t = \dfrac{225 - 625}{160} = \mathbf{-2.5\ m/s^2}$

Exercise 3.3 (b)

3.3 A car enters a curve of radius 60 m at 25 m/s. It brakes steadily, and is at 15 m/s after 80 m along the curve. The tyres can give a total acceleration of at most $0.80g$.

(b) Find the size of the total acceleration as the car enters the curve. [2]

Exercise 3.3 (b)

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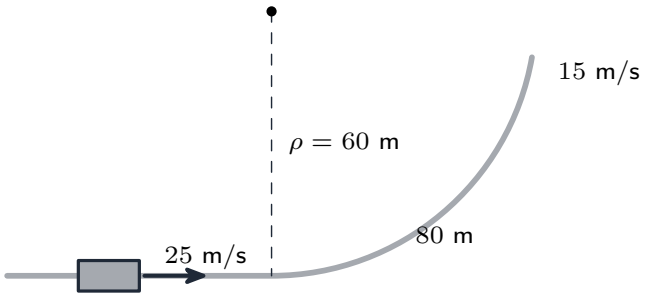
(b) Find the size of the total acceleration as the car enters the curve. [2]

(b) $a_n = \dfrac{25^2}{60} = 10.4 \text{ m/s}^2$; $|\vec{a}| = \sqrt{10.4^2 + 2.5^2} = \mathbf{10.7 \text{ m/s}^2}$

Exercise 3.3 (c)

3.3 A car enters a curve of radius 60 m at 25 m/s. It brakes steadily, and is at 15 m/s after 80 m along the curve. The tyres can give a total acceleration of at most $0.80g$.

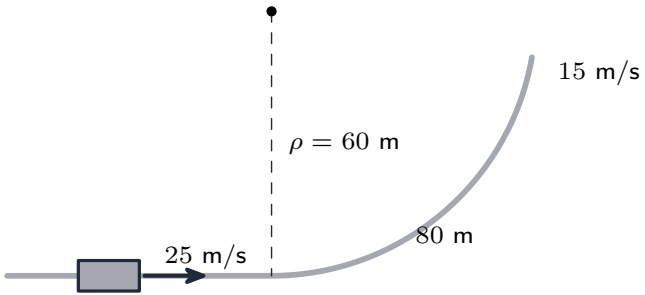
(c) Decide whether the car skids. If it does, find the highest entry speed that is safe when the car brakes at the same a_t. [4]



Exercise 3.3 (c)

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(c) Decide whether the car skids. If it does, find the highest entry speed that is safe when the car brakes at the same a_t. [4]



(c) limit $0.80 \times 9.81 = 7.85$ m/s^2 < 10.7: **the car skids**; $a_n \le \sqrt{7.85^2 - 2.5^2} = 7.44$ m/s^2; $v \le \sqrt{7.44 \times 60} =$ **21.1 m/s**, about 76 km/h, so the driver must slow down before the curve, not on it