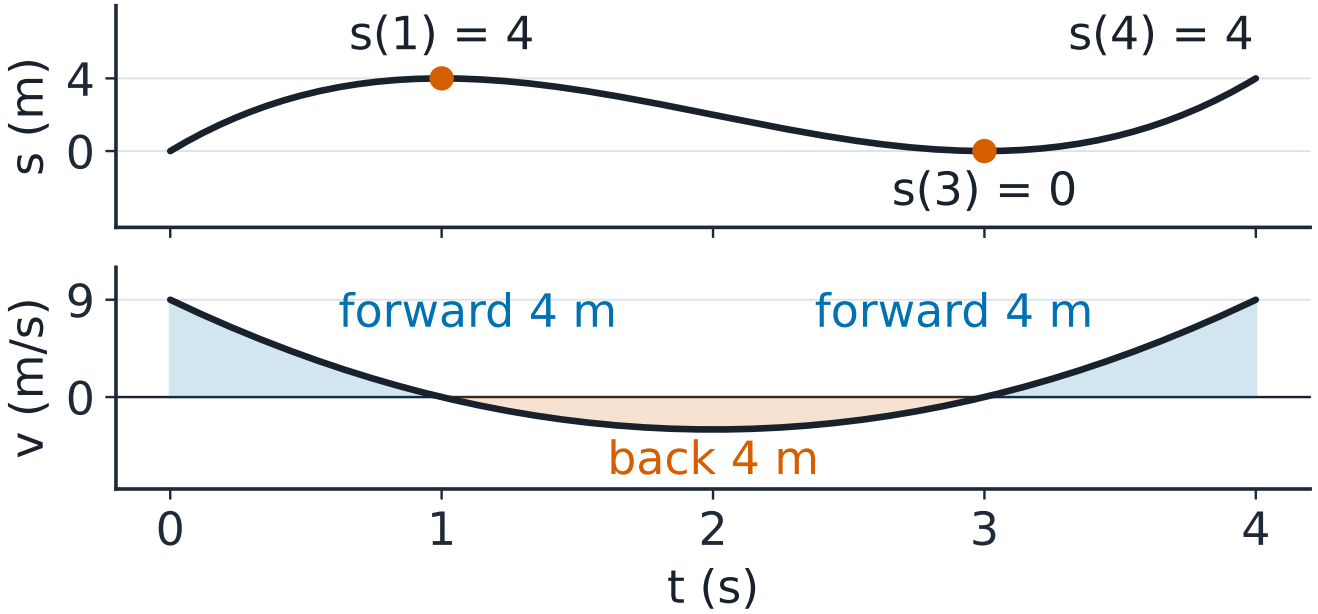
Exercise walk-through

2.2
Linear algebra

University Mechanical Engineering

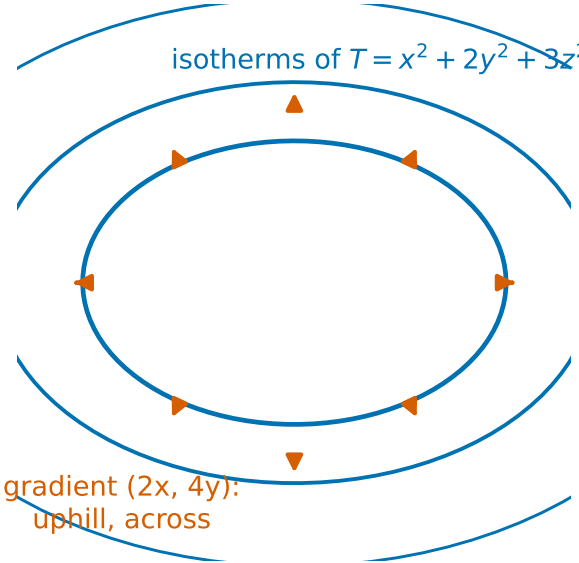
Calculus: the trace is a dictionary



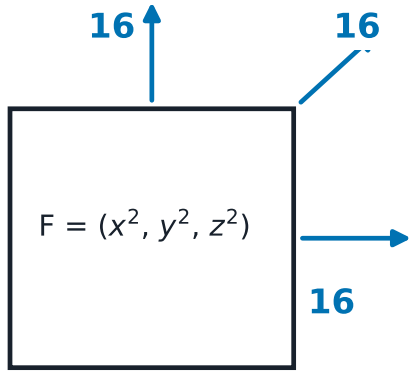
Displacement 4 m but distance **12 m**: v changes sign at t = 1 and 3.

Vector calculus: count what leaves

Fields: T rises at 7.483 K/m; q = -40 grad T



Flux 48, counted two ways



surface flux: 16 + 16 + 16 = 48

divergence: 2(x+y+z) dV = 48

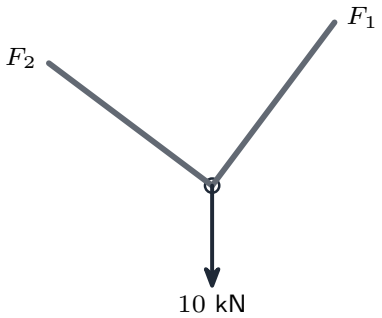
∇T gives **7.483** K/m; **flux 48** by surface = **48** by **divergence**.

The project sheet, reproducible

1. **Motion Displacement** 4 m, distance 12 m; fillet volume $\pi \times 17.067 = 53.62$ cm^3.
2. **Algebra** Truss joint (2.857, 3.571) kN; eigenvalues 1, 3 with modes (1, 1), (1, -1); condition numbers 3 and 250,008.
3. **Engine** ω_n $= 5$, $\zeta = 0.2$, ω_d $= 4.899$; first minimum -0.527; M(1) $= 2.5$, peak 2.552 at r $= 0.959$. Euler $\div 2$, trapezoid $\div 4$, Simpson $\div 15.9$. **Flux** $48 = 48$; circulation 2π; $\nabla^2 \mathrm{T} = 12$. u_c $= 0.0042$, U $= 0.008$, A $= 490.9 \pm 0.3$ mm^2.

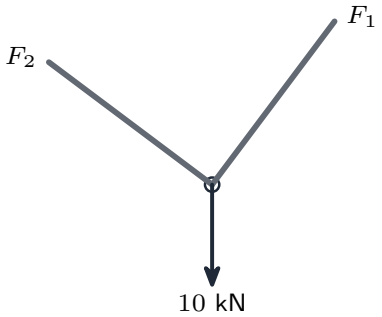
Exercise 2.1

2.1 At a truss joint, $0.6F_1 - 0.8F_2 = 0$ and $0.8F_1 + 0.6F_2 = 10$ (kN). Solve for F_1 and F_2 with the inverse matrix. [3]



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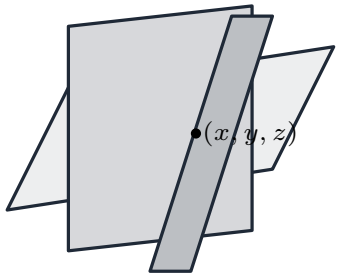


2.1

$$\det = 0.36 + 0.64 = 1;\ \vec{F} = \begin{pmatrix} 0.6 & 0.8 \\ -0.8 & 0.6 \end{pmatrix} \begin{pmatrix} 0 \\ 10 \end{pmatrix} = (\mathbf{8.0},\ \mathbf{6.0})\ \mathbf{kN}$$

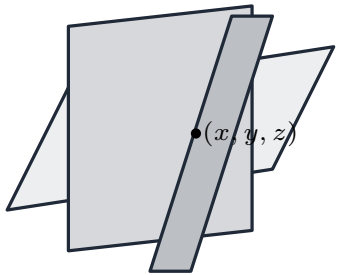
Exercise 2.2

2.2 Solve by elimination: $x + y + z = 6$, $2x - y + z = 3$, $x + 2y - z = 2$. [4]



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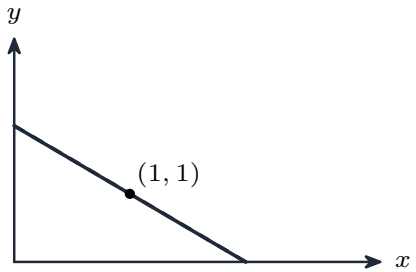
2.2

$R_2 - 2R_1$: $-3y - z = -9$; $R_3 - R_1$: $y - 2z = -4$

from these: $y = 2$, $z = 3$; then $x = 6 - 2 - 3 = \mathbf{1}$: $(x, y, z) = (\mathbf{1}, \mathbf{2}, \mathbf{3})$

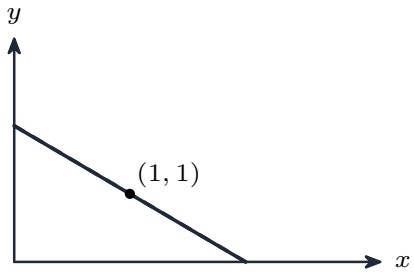
Exercise 2.3

2.3 The system $x + y = 2$, $x + 1.001y = 2.001$ has the solution $(1, 1)$. The right-hand side 2.001 is a measurement; it is changed to 2.002. Solve again, and explain the result using $\det A$. [3]



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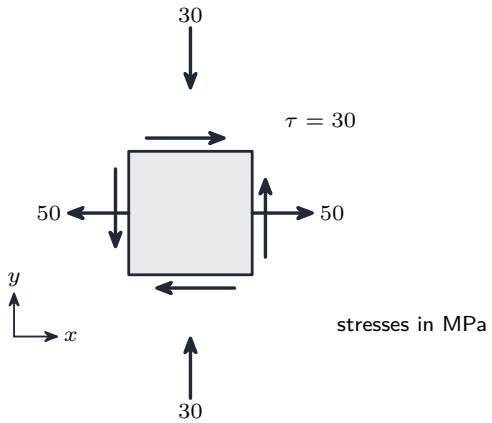
2.3

$0.001y = 0.002$, so $y = 2$, $x = \mathbf{0}$: the answer is $(0,\ 2)$

a 0.05% change in the data moved x from 1 to 0; $\det A = 0.001$ is tiny: the lines are almost parallel, the system is **ill-conditioned**

Exercise 2.4

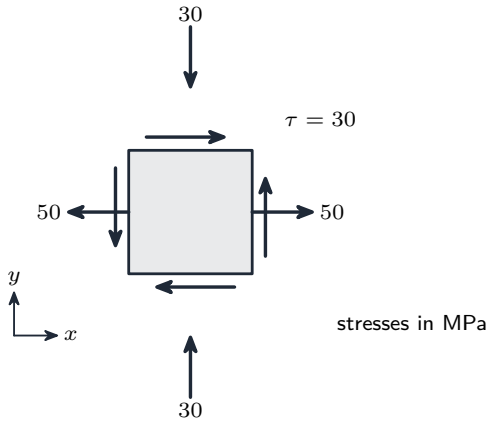
2.4 At a point in a bracket, $\sigma_x = 50$ MPa, $\sigma_y = -30$ MPa and $\tau = 30$ MPa. Find the principal stresses as the eigenvalues of the stress matrix, and the angle θ_p of the first principal direction from x, using $\tan 2\theta_p = \dfrac{2\tau}{\sigma_x - \sigma_y}$. [3]



stresses in MPa

Exercise 2.4

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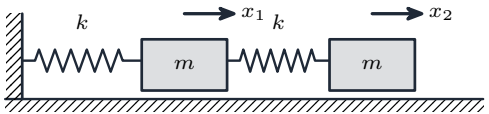
2.4

$\lambda^2 - 20\lambda - 2400 = 0$; $\lambda = 10 \pm 50$: **60 MPa** and **−40 MPa**; $\tan 2\theta_p = 0.75$, $\theta_p = \mathbf{18.4}°$

Exercise 3.1

3.1 Two carts, each of mass $m = 2.0$ kg, are joined to a wall and to each other by springs of stiffness $k = 800$ N/m: wall, spring, cart 1, spring, cart 2. The stiffness matrix is $K = \begin{pmatrix} 2k & -k \\ -k & k \end{pmatrix}$.

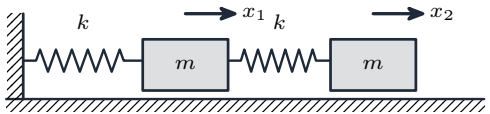
Find the two natural frequencies in Hz and the mode shapes x_2/x_1. Describe how the carts move in each mode. [6]



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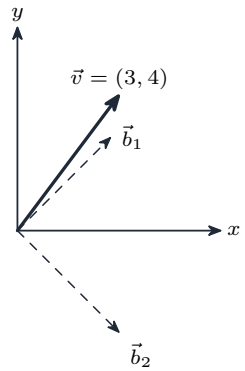
$$\frac{K}{m} = \begin{pmatrix} 800 & -400 \\ -400 & 400 \end{pmatrix};\ \lambda^2 - 1200\lambda + 160\,000 = 0$$

$\omega^2 = 152.8$ and 1047 rad^2/s^2; $f = \dfrac{\omega}{2\pi} =$ **1.97 Hz** and **5.15 Hz**

mode 1: $x_2/x_1 =$ **1.62**: both carts move the same way, cart 2 further; mode 2: $x_2/x_1 =$ **−0.62**: they move in opposite directions

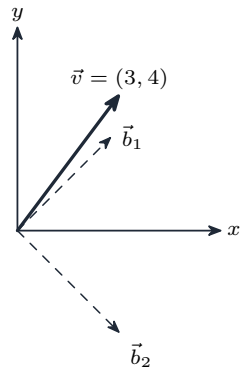
Exercise 3.2

3.2 A sensor gives the vector $\vec{v} = (3,\ 4)$ in the x, y basis. Write $\vec{v}$ in the basis $\vec{b}_1 = (1, 1)$, $\vec{b}_2 = (1, -1)$. Then explain why the basis $\hat{e}_1 = \frac{1}{\sqrt{2}}(1, 1)$, $\hat{e}_2 = \frac{1}{\sqrt{2}}(1, -1)$ is easier to use. [4]



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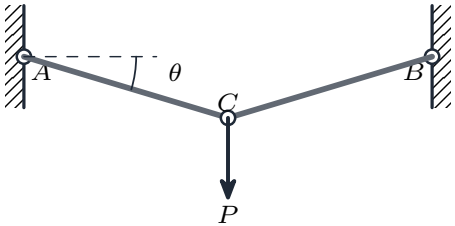
$a + b = 3$, $a - b = 4$: $a = \mathbf{3.5}$, $b = \mathbf{-0.5}$

$\hat{e}_1$, $\hat{e}_2$ are orthonormal (perpendicular, length 1), so each coefficient is a dot product: $\vec{v} \cdot \hat{e}_1 = 7/\sqrt{2}$, with no equations to solve

Exercise 3.3 (a)

3.3 Two equal bars meet at a joint C and carry a vertical load $P = 10$ kN. Each bar makes an angle θ with the horizontal. Joint equilibrium gives $\begin{pmatrix} \cos\theta & -\cos\theta \\ \sin\theta & \sin\theta \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = \begin{pmatrix} 0 \\ P \end{pmatrix}$.

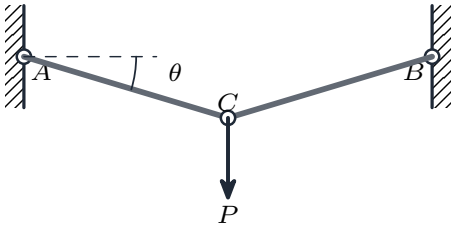
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(a) $\det = 2\sin\theta\cos\theta = \sin 2\theta = 0.866$; $F_1 = F_2 = \dfrac{P}{2\sin\theta} = \mathbf{10\ kN}$

Exercise 3.3 (b)

3.3 Two equal bars meet at a joint C and carry a vertical load $P = 10\ \text{kN}$. Each bar makes an angle θ with the horizontal. Joint equilibrium gives $\begin{pmatrix} \cos\theta & -\cos\theta \\ \sin\theta & \sin\theta \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = \begin{pmatrix} 0 \\ P \end{pmatrix}$.

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(b) $F = \dfrac{10}{2\sin 2.0°} = \mathbf{143\ kN}$; $\det = \sin 4° = \mathbf{0.070}$

Exercise 3.3 (c)

3.3 Two equal bars meet at a joint C and carry a vertical load $P = 10\ \text{kN}$. Each bar makes an angle θ with the horizontal. Joint equilibrium gives $\begin{pmatrix} \cos\theta & -\cos\theta \\ \sin\theta & \sin\theta \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = \begin{pmatrix} 0 \\ P \end{pmatrix}$.

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(c) $F = \dfrac{10}{2\sin 2.1^\circ} = 136$ kN, a change of **4.8**% for a 5% angle error; the system is ill-conditioned when θ is small, so keep θ well away from 0 (for example above 15°)