

## Communication Systems

Curated undergraduate study track (not accredited, no external exam). Original explanations and practice only, following the reviewed course handout.

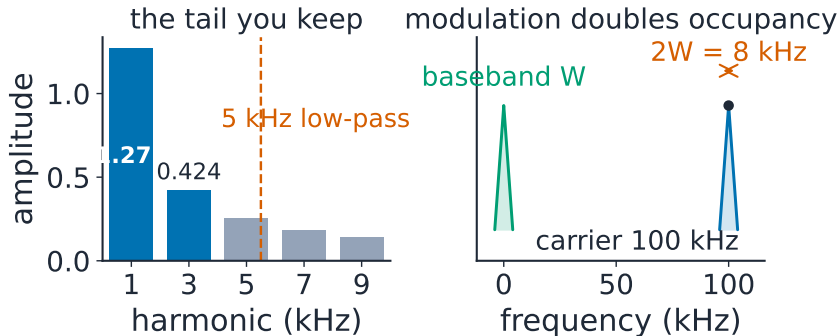
## Sending signals across real channels

- Analogue first (spectra, AM, FM), then the digital chain (noise, PCM, modulation), then the limits (**entropy**, capacity, coding).
- Every dB claim recomputed as a ratio; every power budget closed; every bandwidth tested against its channel.

## Spectra and bandwidth: the tail you keep

- 1 Harmonics** Amplitudes 1.273, 0.424, 0.255 - a 5 kHz low-pass on a 1 kHz square keeps the 1st and 3rd, and the wave rounds over.
- 2 Power** The odd-harmonic power series sums to  $\pi^2/8$ , so the fundamental holds  $8/\pi^2 = 0.811$  of everything - energy concentrates low.
- 3 Channel** A 4 kHz message on a 100 kHz carrier fills 96-104 kHz; single-sideband sends only the top half and halves the spectrum.

# The tail you keep

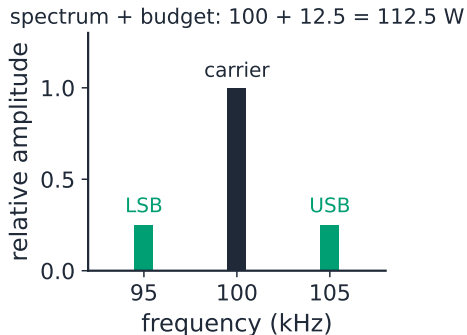
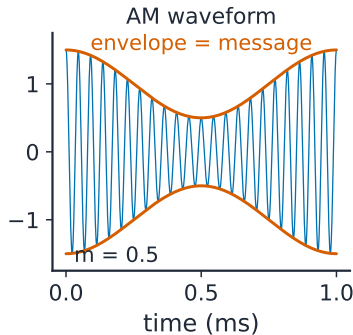


A 1 kHz square through a 5 kHz low-pass keeps harmonics 1 and 3 (about 90 percent of the power); modulation doubles occupancy to  $2W$ .

## Amplitude modulation: the power budget

- 1 **Budget** Carrier 100 W,  $m = 0.5$ : sidebands 12.5 W, total 112.5 W, 11.1% efficient; at  $m = 1$  it is  $50/150 \text{ W} = 33.3\%$  - two-thirds of the transmitter heats the antenna.
- 2 **Detector** Carrier 100 kHz, message 5 kHz:  $\tau$  between 1.6 and 31.8  $\mu\text{s}$ ; 10  $\mu\text{s}$  fits. At  $m = 0.9$  the upper bound tightens to 13.9  $\mu\text{s}$ .
- 3 **Fix** DSB-SC puts ALL the transmitter power into sidebands (the  $m = 0.7$  station of sheet 11.2: 2450 W becomes 12 450 W,  $5.1\times$ ) - the receiver pays with a coherent carrier.

## The envelope and the budget



At  $m = 0.5$  the sidebands carry 12.5 W of 112.5 W - 11 percent; the carrier takes the rest.  $100 \text{ W} + 12.5 \text{ W}$  closes on  $\text{carrier} \times (1 + m^2/2)$ .

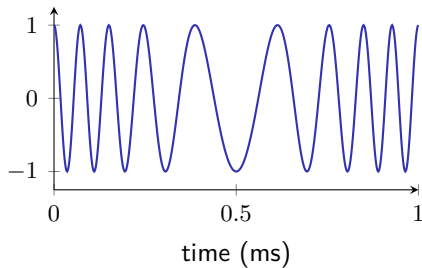
## Angle modulation: Carson draws the line

- 1 **Broadcast**  $\Delta f = 75$  kHz,  $f_m = 15$  kHz:  $\beta = 5$ ,  $B = 180$  kHz inside the 200 kHz channel - the guard band is designed in.
- 2 **Range**  $\Delta f = 50$  kHz: at  $f_m = 15$  kHz  $B = 130$  kHz; at 300 Hz  $B = 101$  kHz - the highest message frequency sets the worst case.
- 3 **Trade** Wideband FM converts bandwidth to SNR at  $3\beta^2/2$  (15.7 dB at  $\beta = 5$ ) - and loses it all below threshold, where clicks and capture rule.

# The frequency bends, Carson fences

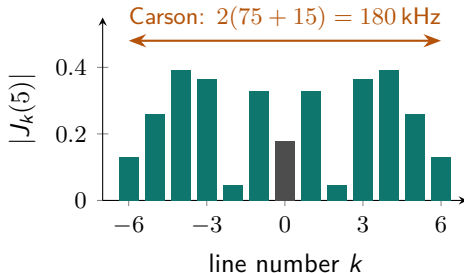
## FM waveform

carrier 9 kHz, message 1 kHz,  $\beta = 5$



## Spectrum: Bessel lines, $\beta = 5$

lines 15 kHz apart

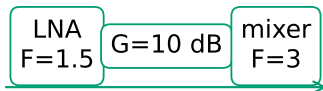


The **frequency** swings with the message; the Bessel lines fit inside the **Carson** 180 kHz fence, so 200 kHz channels keep a guard.

## Noise and SNR: the front end decides

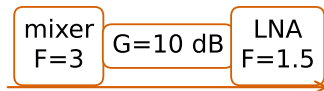
- 1 **Cascade** LNA  $F = 1.5$ ,  $G = 10$  dB before mixer  $F = 3$ :  $F = 1.70$  (2.3 dB); reversed, 3.05 (4.84 dB) - the order is worth 2.5 dB.
- 2 **Floor**  $-174 + 53 = -121$  dBm in a 200 kHz channel; add NF 2.3 and demand 10 dB SNR: the minimum signal is -108.7 dBm.
- 3 **Cliff** BPSK bit energy at 5 dB:  $6 \times 10^{-3}$ ; 9 dB:  $3.3 \times 10^{-5}$ ; 12 dB:  $9 \times 10^{-9}$  - each dB near the wall buys an order of magnitude.

## Same blocks, swapped order

**LNA → mixer**

$$F = 1.5 + (3-1)/10$$

$$= 1.70 \text{ (2.30 dB)}$$

**mixer → LNA**

$$F = 3 + (1.5-1)/10$$

$$= 3.05 \text{ (4.84 dB)}$$

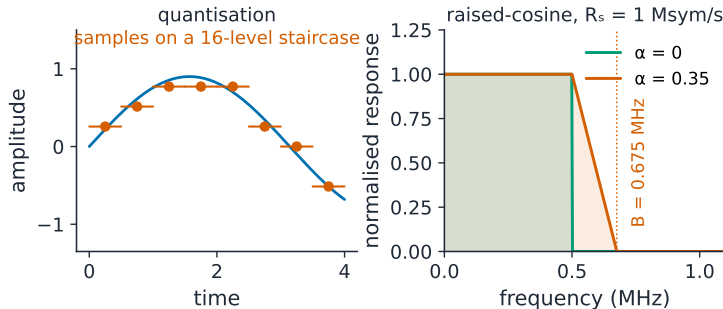
thermal floor:  $-174 \text{ dBm/Hz} + 10 \cdot \log_{10} B$  ( $-121.4 \text{ dBm}$  in  $180 \text{ kHz}$ )

LNA first:  $F = 1.70$  (2.30 dB). Mixer first: 4.84 dB - front-end gain IS the protection, above the  $-121.4 \text{ dBm}$  floor.

## PCM and baseband: bits from waves

- 1 Voice** 8 kHz sampling  $\times$  8 bits = 64 kbps at 49.9 dB - the trunk standard; 12-bit masters buy 74.0 dB.
- 2 Design** 60 dB target needs  $n = 10$  (61.96 dB): 44.1 kHz  $\times$  10 = 441 kbps, 3.31 MB per minute.
- 3 Fit** 1 Msym/s in a 0.75 MHz channel:  $\alpha = 0.5$  lands exactly at 0.75 MHz; Manchester coding would need twice that and does not fit - NRZ plus a scrambler it is.

# From voltage to shaped pulses



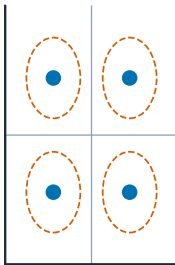
Samples on a 16-level staircase (SQNR  $6.02n + 1.76$  dB) and the **raised-cosine** 0.675 MHz at  $\alpha = 0.35$ .

## Digital modulation: the constellation plane

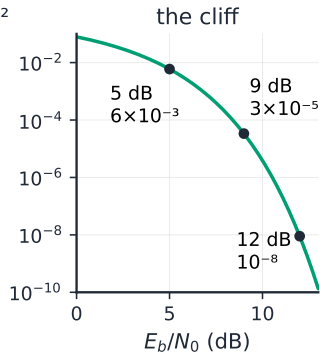
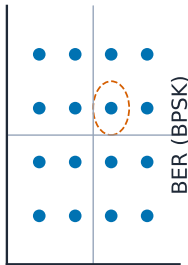
- 1 **Energy**  $d = 2$ : QPSK averages  $E = 2$ , 16-QAM averages 10 - a factor 5 = 7.0 dB for twice the bits per symbol.
- 2 **Rates** 2 Mbps with  $\alpha = 0.35$ : QPSK needs 0.675 MHz (2.96 b/s/Hz); 16-QAM needs 0.3375 MHz (5.93 b/s/Hz) - both under Shannon's 6.66 at 20 dB.
- 3 **Bits** QPSK at the same symbol energy halves the bit energy: 5 dB drops to 2 dB and BER goes  $6 \times 10^{-3}$  to  $3.7 \times 10^{-2}$  - bits, not symbols, consume energy.

# Points, distance and the cliff

QPSK:  $E = d^2/2$



16-QAM:  $E = 2.5 d^2$



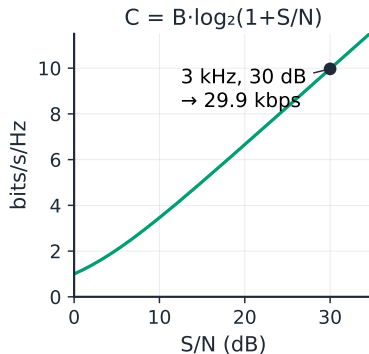
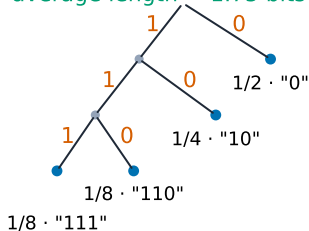
Equal  $d$ : QPSK  $d^2/2$ , 16-QAM  $2.5d^2$  (7 dB for  $2 \times$  bits); BER falls two decades per 4 dB - a cliff, not a slope.

## Information and coding: the limits

- 1 **Source** Quartile source ( $1/2, 1/4, 1/8, 1/8$ ):  $H = 1.75$  bits and Huffman lengths 1,2,3,3 average exactly 1.75 - dyadic probabilities meet the bound.
- 2 **Channel** 3 kHz at 30 dB:  $C = 29.9$  kbps - the dial-up ceiling, approached (V.90) and never beaten.
- 3 **Code** Encode 1011 to 0 1 1 0 0 1 1; flip bit 5 and the syndrome reads  $101_2 = 5$  - the code points at its own error. Doubles miscorrupt silently unless extended (8,4) catches them.

# The bound met, the ceiling drawn

average length = 1.75 bits =  $H$



Huffman lengths 1,2,3,3 average exactly  $H = 1.75$  bits; capacity 3 kHz at 30 dB is 29.9 kbps - approached never beaten.

## Check yourself

- AM: carrier 100 W,  $m = 1$  - **sideband** power and efficiency? (50 W, 33.3%.)
- FM:  $\Delta f$  75 kHz,  $f_m$  15 kHz - Carson bandwidth? (180 kHz.)
- PCM: 8 bits - SQNR? (49.9 dB.)
- Coding: quartile source **entropy**? (1.75 bits.)