

Control Systems

Curated undergraduate study track (not accredited, no external exam). Original explanations and practice only, following the reviewed course handout.

Closing the loop

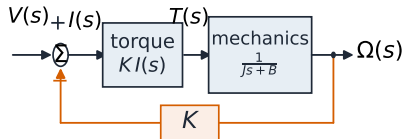
- EE06 wrote the open-loop system function; this course closes the loop and asks how to make the output follow the input: fast, accurate, without ringing.
- Every design is checked in both domains: a margin predicts a damping, a pole placement predicts an **overshoot**, and Routh, locus and Nyquist must agree.

Modelling: two laws, one loop

- 1 **RC** $R = 2 \text{ k}\Omega$, $C = 100 \text{ }\mu\text{F}$: $\tau = 0.2 \text{ s}$; a 5 V step reaches 3.16 V (63.2 percent) at one **time constant**.
- 2 **Motor** $J = 0.01$, $B = 0.1$, $K = 0.5$, $R = 1$: $\omega/V = K \text{ over } (Js+B)R + K \text{ squared} = 50/(s+35)$; $\tau = 28.6 \text{ ms}$, $1.43 \text{ rad/s per volt}$.

The motor as it is wired

torque law Newton
 $T = Ki$ $J\dot{\omega} = T - B\omega$



KVL: $V = Ri + K\omega$

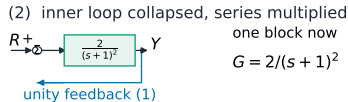
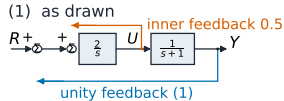
with $L \approx 0$:
$$\frac{\Omega(s)}{V(s)} = \frac{K}{(Js+B)R + K^2} = \frac{50}{s+35}$$

Torque law and Newton close through the **back-EMF** into one inner loop.

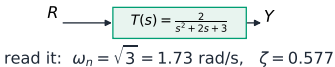
Transfer functions and block diagrams

- 1 **One loop** $G_1 = 1/(s+1)$, $G_2 = 1/(s+2)$, $K = 6$, unity **feedback**: $T = 6/(s^2 + 3s + 8)$; $\omega_n = 2.83$, $\zeta = 0.530$.
- 2 **Nested** Inner $2/s$ with 0.5 collapses to $2/(s+1)$; series with $1/(s+1)$; outer unity gives $2/(s^2 + 2s + 3)$; $\zeta = 0.577$.

Three reductions, one picture



(3) close the outer loop: $T = G/(1+G)$

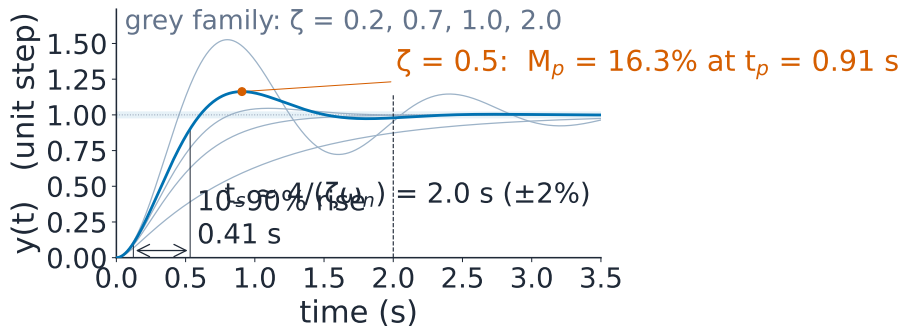


Inner loop first, series second, outer loop last; each panel is one rule.

Time-domain response: the second-order dictionary

- 1 **Specs** $\omega_n = 4$, $\zeta = 0.5$: **overshoot** 16.3 percent, peak 0.907 s, settling 2.0 s (2 percent band), rise 0.41 s.
- 2 **Design** K over $s(s+2)$ with $\zeta = 0.45$: $K = 4.94$, $K_v = 2.47$, ramp error 0.405, step error exactly zero.

One damping ratio, one shape

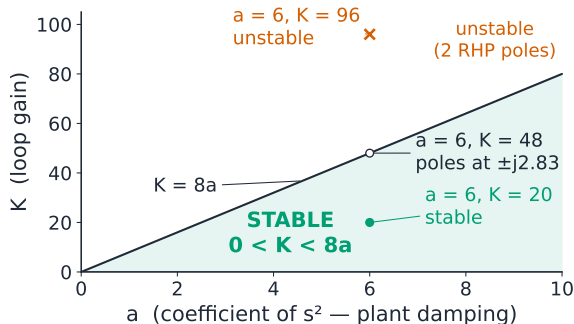


The $\zeta = 0.5$ curve annotated with **overshoot**, peak, band and rise.

Routh: count, do not solve

- 1 **Quartic** $s^4 + 2s^3 + 3s^2 + 4s + 5$: first column 1, 2, 1, minus 6, 5: two sign changes, two RHP roots (exact: plus 0.29 plus-minus $j1.42$).
- 2 **Range** $s^3 + 5s^2 + 6s + K$: the s^1 row is $(30 \text{ minus } K)/5$, so $0 < K < 30$; at $K = 30$ the auxiliary gives plus-minus $j2.45$.

The stable design space

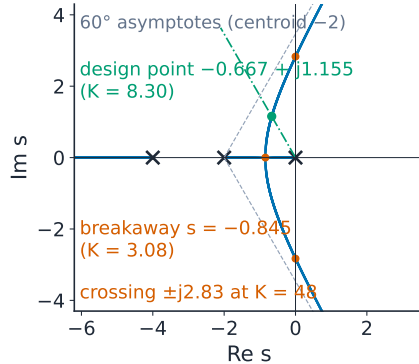


For $s^3 + a s^2 + 8 s + K$, stability is exactly $0 < K < 8a$, a straight boundary.

The locus of one knob

- 1 **Sketch** K over $s(s+2)(s+4)$: centroid minus 2, asymptotes 60 degrees, breakaway minus 0.845 at $K = 3.08$, crossing plus-minus $j2.83$ at $K = 48$.
- 2 **Design** $\zeta = 0.5$ ray meets the locus at minus $0.667 + j1.155$ with $K = 8.30$; the third pole minus 4.67 stays well left.

Poles leave, meet and bend

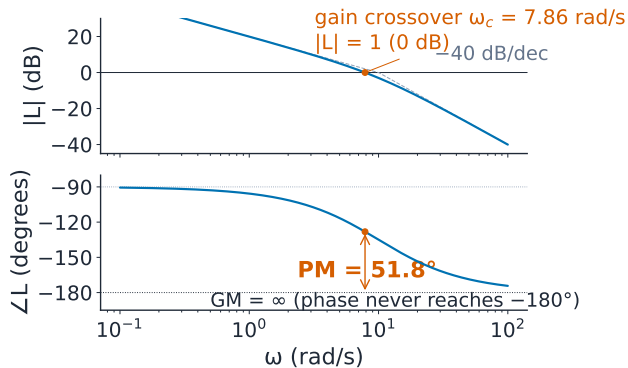


Breakaway at minus 0.845, asymptotes from minus 2, jw crossing at plus-minus $j2.83$ ($K = 48$).

Margins: the engineer's tolerance

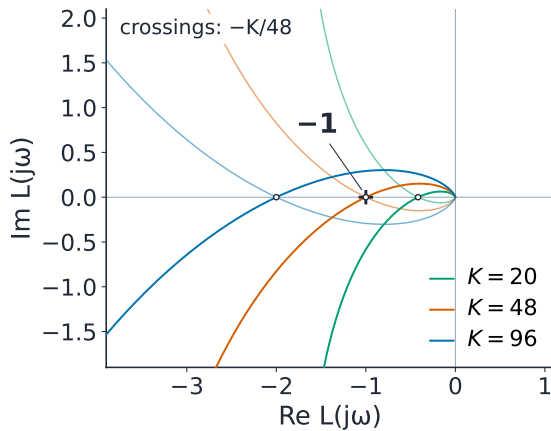
- 1 **PM by arithmetic** 100 over $s(s+10)$: crossover 7.86 rad/s, phase minus 128.2 degrees, PM = 51.8 degrees, GM infinite; exact closed loop zeta = 0.5.
- 2 **Three ways** 20 over $s(s+2)(s+4)$: GM = 2.4 (7.6 dB), PM = 25.7 degrees at 1.73 rad/s; Nyquist wraps twice at $K = 96$: $Z = 2$, Routh agrees.

Bode and Nyquist analysis: straight lines with a verdict



Read PM at the **gain crossover**; the phase never reaches minus 180, so GM is infinite.

One shape slides through minus one

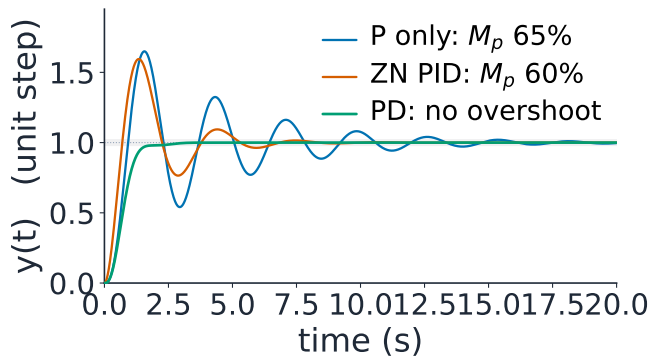


Gain scales the curve; $K = 48$ passes through minus one, $K = 96$ wraps twice.

PID: three terms, three prices

- 1 **ZN** $K_u = 48$, $T_u = 2.22$ s: $K_p = 28.8$, $T_i = 1.11$ s, $T_d = 0.278$ s; P-only rings 65 percent, the ZN PID rings 60 percent and removes the ramp error.
- 2 **Retune** Same K_p with $T_d = 0.5$ s: poles minus 2 and minus 2 plus-minus $j3.22$, no **overshoot**, settling 2.1 s (**derivative** on the measurement; on the error it rings 14 percent); the ramp error stays 0.069.

One plant, three tunings

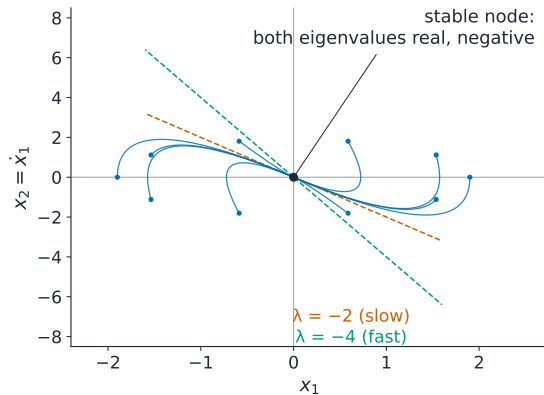


P only overshoots 65 percent; Ziegler-Nichols rings at 60 with zero ramp error; the PD retune (**derivative** on the measurement) shows none.

State space: poles by algebra

- 1 **Decay** 8 over $(s+2)(s+4)$: $\Phi(0.5)$ has entries 0.600 , 0.116 , minus 0.930 , minus 0.097 and eigenvalues e to the minus 1 and e to the minus 2 .
- 2 **Place** Both poles at minus 10 : matching $s^2 + 20s + 100$ gives $K_{fb} = [92, 14]$; the eigenvalues of A minus BK land exactly on minus 10 .

Trajectories straighten onto the slow pole



A stable node: every trajectory bends onto the $\lambda = -2$ eigenvector.

Check yourself

- Derive the motor model from its two laws and name its **time constant** and DC gain.
- Reduce a nested **block diagram** to one fraction and read the **natural frequency** and damping from the denominator.
- Count RHP roots with the array, price a damping ray in gain, and read both margins off a Bode sketch.
- Tune the three PID terms against their price list, and place both poles of a two-state model by algebra.