

Signals and Systems

Curated undergraduate study track (not accredited, no external exam). Original explanations and practice only, following the reviewed course handout.

What does a system do to ANY input

- EE04 asked what one circuit does at one frequency; this course answers for any input.
- Convolution, Fourier, and z - the three answers the rest of the degree runs on.

System properties: test from the definition

- 1 **Trap** $y = x(2t)$ is time-VARYING: a shifted input gives a half shift ($u(2t-1) = u(t-1/2)$), not the full one.
- 2 **Test** Compare shift-then-system against system-then-shift - the one-line test every property question reduces to.
- 3 **Stability** BIBO for LTI: integral of $|h|$ finite - the integrator fails, the exponential passes.

Four tests, four verdicts

linearity

feed $x_1 + x_2$
output = $y_1 + y_2$?

time invariance

shift x by T
output shifts by T only?

sum in, sum out

shift in, shift out

causality

output at $t = 0$
depends on $x(t > 0)$?

BIBO stability

bounded x in
bounded y out?

no output before input

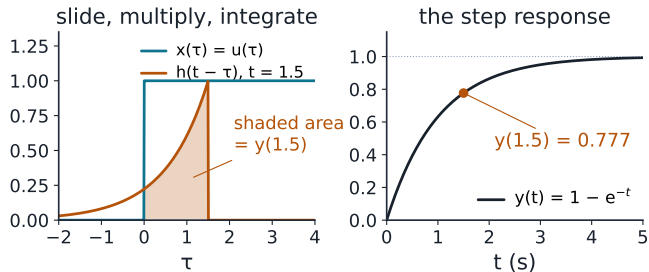
integral $|h| < \text{infinity}$

Sum in sum out; shift in shift out; no output before input; bounded in bounded out - each property is one probe experiment.

Convolution: superposition made mechanical

- 1 **Step** $u * e^{-t} u = 1 - e^{-t}$: the first-order step response, both endpoints check (0 at 0, 1 at infinity).
- 2 **Widths** Two width-2 rectangles convolve to a width-4 triangle peaking at 2.
- 3 **Area** $e^{-2t} * e^{-3t} = e^{-2t} - e^{-3t}$; its integral $1/2 - 1/3 = 1/6$ equals the product of the input areas.

Slide, multiply, integrate

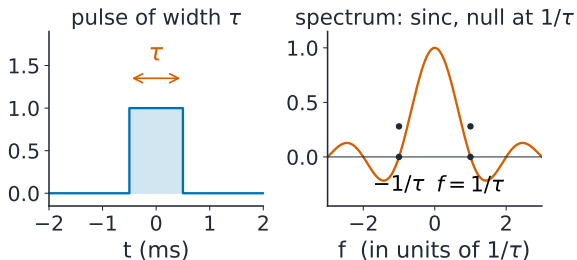


The flipped h slides past x ; each output value is the overlap area - $y(1.5) = 0.777$ for the step into e^{-t} .

Fourier: convolution becomes multiplication

- 1 **Pair** Width-tau rectangle transforms to $\tau \text{sinc}(\omega \tau/2)$: value at 0 is the area, first null at $f = 1/\tau$.
- 2 **Shift** $x(t-3)$ maps to $X e^{-j3\omega}$; modulation moves the spectrum to the carrier.
- 3 **Response** $H = 1/(1+j\omega)$ at $\omega = 1$: gain $1/\sqrt{2}$, phase -45 degrees - the EE05 corner, found by transforms.

Pulse to sinc, null at $1/\tau$



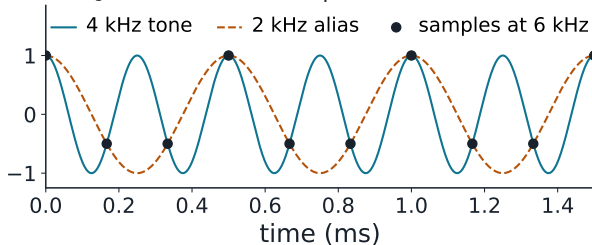
A rectangle in time transforms to a sinc in frequency; the 1 ms click spans to 1 kHz, the 0.1 ms click to 10 kHz.

Sampling: replicas, Nyquist, and the fold

- 1 **Alias** 4 kHz at 6 kHz sampling: $f_s/2 = 3$ kHz, so the tone reads as $6 - 4 = 2$ kHz.
- 2 **Verify** $\cos(2\pi 4n/6)$ and $\cos(2\pi 2n/6)$ agree sample by sample - the even-cos identity.
- 3 **Fix** 44.1 kHz audio with 38 kHz interference: the alias lands at 6.1 kHz; filter before sampling or oversample.

Two tones, one sequence

$f = 4 \text{ kHz} > f_s/2 = 3 \text{ kHz}$: the samples cannot tell 4 kHz from 2 kHz

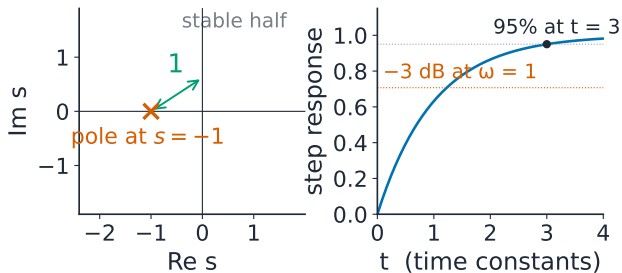


Sampled at 6 kHz, a 4 kHz cosine and a 2 kHz cosine pass through exactly the same samples - the fold is complete.

System functions: poles tell the story

- 1 **One pole** $1/(s+1)$: $h = e^{-t} u$; step $1 - e^{-t}$; at $j\omega$ 1, gain $1/\sqrt{2}$ - three views, one system.
- 2 **Resonance** $s/(s^2+2s+101)$: poles $-1 \pm j10$, a band-pass peaking near ω 10 with gain about 0.5.
- 3 **Feedback** $K/((s+1)(s+2))$ closed loop: second-order stability for $K > -2$; at $K=2$ the poles sit at $-1.5 \pm j1.32$.

One pole, three readings

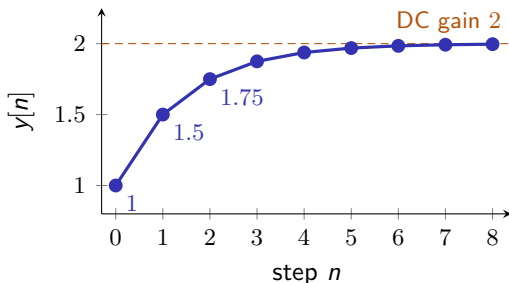


The pole at $s = -1$ is the decay e^{-t} , the -3 dB point at $\omega = 1$, and the 95% settle at $t = 3$ - one number, three readings.

Discrete systems: roots inside the unit circle

- 1 **Accumulator** $y = 0.5y + x$: step response 1, 1.5, 1.75, 1.875 - converging to the DC gain 2.
- 2 **Oscillator** $y = -0.25y[n-2] + x$: roots $\pm j0.5$, a period-4 mode under a 0.5^n envelope; DC gain 0.8.
- 3 **Averager** 3-point mean: DC gain 1, alternating gain $1/3$ - attenuation exactly as the sum predicts.

The gap halves each step



$$y_{\infty} = \frac{1}{1 - 0.5} = 2$$

DC gain

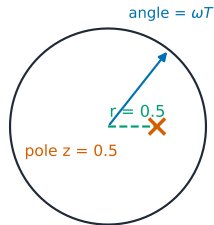
n	$y[n]$	gap to 2
0	1	1
1	1.5	0.5
2	1.75	0.25
3	1.875	0.125

$y[n] = 0.5 y[n - 1] + x[n]$ on a unit step: the **gap** to 2 halves every step.

The z-transform: Laplace wrapped around

- 1 **Accumulate** $1/(1 - 0.5 z^{-1})$: step response $2(1 - 0.5^{n+1})$ - matches the iteration at $n = 0$ and at infinity.
- 2 **Differencer** $(1 + z^{-1})/(1 - 0.5z^{-1})$ on a step: $y[n] = 4 - 3(0.5)^n$; iteration confirms 1 then 2.5.
- 3 **High-pass** $(1 - z^{-1})/2$: zero at DC, one at Nyquist, $\sin(\omega/2)$ in between - the anti-averager.

Frequency as angle, damping as radius



the circle is the frequency axis; radius 0.5 = damping -693 s^{-1} at $T = 1 \text{ ms}$

The pole at $z = 0.5$ with $T = 1 \text{ ms}$ is a decay of -693 per second - the two planes are one subject.

Check yourself

- Properties: is $y = x(2t)$ time-invariant? (No - half shift.)
- Sampling: 4 kHz at 6 kHz reads as? (2 kHz.)
- z-plane: pole at 0.5 means? (Stable - inside the **unit circle**.)