Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

Ampère on a circle of radius r: $B \cdot 2\pi r = \mu_0 I$; $B = \dfrac{4\pi \times 10^{-7} \times 15}{2\pi \times 0.20} = \mathbf{15\ \mu T}$; about **30**% of the Earth's field, enough to upset a compass

2.2

$\dfrac{F}{L} = \dfrac{\mu_0 I_1 I_2}{2\pi d} = \dfrac{2 \times 10^{-7} \times 500 \times 500}{0.10} = \mathbf{0.50\ N/m}$; opposite currents **repel**

2.3

$evB = \dfrac{mv^2}{r}$, so $r = \dfrac{mv}{eB} = \dfrac{9.11 \times 10^{-31} \times 2.0 \times 10^{6}}{1.60 \times 10^{-19} \times 0.010} = \mathbf{1.14\ mm}$; $T = \dfrac{2\pi m}{eB} = \mathbf{3.6\ ns}$

2.4

$B(0) = \dfrac{\mu_0 I}{2R} = \dfrac{4\pi \times 10^{-7} \times 5.0}{0.080} = \mathbf{78.5\ \mu T}$; $B = \dfrac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}}$ with $R^2 + z^2 = (5.0\text{ cm})^2$;
$78.5 \times \dfrac{4^3}{5^3} = \mathbf{40.2\ \mu T}$

3.1

a circle of radius r inside the core encloses NI: $B = \dfrac{\mu_0 NI}{2\pi r}$; $B = \mathbf{1.20\ mT}$ at 5.0 cm and **0.86 mT** at 7.0 cm

$\Phi = \int_a^b \dfrac{\mu_0 NI}{2\pi r} h\, dr = \dfrac{\mu_0 NIh}{2\pi} \ln \dfrac{b}{a} = 4.04 \times 10^{-7}$ Wb; $L = \dfrac{N\Phi}{I} = \mathbf{54\ \mu H}$

mean radius 6.0 cm: $B = 1.00$ mT and $L = 53.3$ µH, about 1% low, because the core is thin compared with its radius

3.2

near side: $\dfrac{2 \times 10^{-7} \times 20 \times 2.0 \times 0.15}{0.040} = \mathbf{3.0 \times 10^{-5}\ N}$, towards the wire (like currents);
far side: $\dfrac{2 \times 10^{-7} \times 20 \times 2.0 \times 0.15}{0.14} = \mathbf{8.6 \times 10^{-6}\ N}$, away

net force $\mathbf{2.1 \times 10^{-5}\ N}$ towards the wire; the short sides' forces cancel

$\Phi = \int_{0.04}^{0.14} \dfrac{\mu_0 I_1}{2\pi r}(0.15)\, dr = \dfrac{\mu_0 I_1 (0.15)}{2\pi} \ln 3.5 = 7.5 \times 10^{-7}$ Wb; $M = \dfrac{\Phi}{I_1} = \mathbf{38\ nH}$

3.3

(a) $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{2 \times 10^{-7} \times 10}{0.0050} = \mathbf{400\ \mu T}$

(b) the neighbour is 15 mm from S_1: $\dfrac{2 \times 10^{-7} \times 3.0}{0.015} = 40$ µT; **10**% of full scale

(c) the trace's field is opposite at S_1 and S_2, so $S_1 - S_2 = 800$ µT, and a uniform field cancels; the neighbour gives $40 - 24 = 16$ µT (15 and 25 mm away), or **2**%

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(d) the ring measures $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc}$; the neighbour is not enclosed, so it adds nothing; a plain ring (a current transformer) works only for ac, so dc needs a Hall element in a gap of the ring