Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

$\nabla \cdot \mathbf{F} = y + z + x$; at P: $2+3+1 = \mathbf{6}$; $\nabla \times \mathbf{F} = (0-y,\ 0-z,\ 0-x) = (\mathbf{-2}, \mathbf{-3}, \mathbf{-1})$ at P

2.2

$\nabla \cdot \mathbf{F} = 1 + 1 + 0 = 2$, so the volume integral is $2 \times 1 = \mathbf{2}$

faces $x = 1$ and $y = 1$ each give $1 \times 1 = 1$ out; the faces $x = 0$, $y = 0$ have zero field normal to them and the z faces have none: total **2**, the same

2.3

OA: $y = 0$, so 0; AB and CO: $d\mathbf{l}$ along y and $F_y = 0$, so 0; BC: $F_x = 1$ with $dx = -1$ in total, so -1; circulation $= \mathbf{-1}$

$(\nabla \times \mathbf{F})_z = \dfrac{\partial F_y}{\partial x} - \dfrac{\partial F_x}{\partial y} = -1$, times the area 1 gives -1

2.4

on the sphere $F = 2$ everywhere and is normal to it: flux $= 2 \times 4\pi(2)^2 = \mathbf{32}\pi$

$\nabla \cdot \mathbf{F} = \dfrac{1}{r^2}\dfrac{d(r^3)}{dr} = 3$; $3 \times \frac{4}{3}\pi(2)^3 = 32\pi$, the same

3.1

$\nabla \cdot \mathbf{F} = 2 - 2 + 2 = 2$; volume integral $2 \times 8 = \mathbf{16}$

face $x = +1$: $F_x = 2$ over area 4, so $+8$; face $x = -1$: $F_x = -2$ with outward normal $-\hat{\mathbf{x}}$, so $+8$; the y faces give -8 each (inward) and the z faces $+8$ each: $8+8-8-8+8+8 = \mathbf{16}$

$\nabla \times \mathbf{F} = \mathbf{0}$; the field flows out along $\pm x$ and $\pm z$ and in along $\pm y$, with no rotation anywhere

3.2

on S_1: $\dfrac{k}{R^2} \times 4\pi R^2 = \mathbf{4}\pi\mathbf{k}$; the field falls as $\dfrac{1}{r^2}$ while the area grows as r^2, so they cancel

$\nabla \cdot \mathbf{F} = \dfrac{1}{r^2}\dfrac{d}{dr}\left(r^2 \dfrac{k}{r^2}\right) = \mathbf{0}$ for $r \neq 0$

the region between S_1 and S_2 has no sources, so S_2 also carries $\mathbf{4}\pi\mathbf{k}$; S_3 encloses no source, so its flux is **0**

3.3

(a) $\dfrac{\partial F_y}{\partial x} = \dfrac{(x^2+y^2) - 2x^2}{(x^2+y^2)^2}$ and $\dfrac{\partial F_x}{\partial y} = -\dfrac{(x^2+y^2) - 2y^2}{(x^2+y^2)^2}$; their difference is $\dfrac{2(x^2+y^2) - 2x^2 - 2y^2}{(x^2+y^2)^2} = 0$; the x and y parts are zero because nothing depends on z

(b) on C_1, $|\mathbf{F}| = \dfrac{1}{a}$ along the path, so $\oint = \dfrac{1}{a} \times 2\pi a = \mathbf{2}\pi$, the same for every a

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(c) C_2 does not enclose the wire, so the curl is zero over the whole disc inside it; by Stokes' theorem the circulation is **0**

(d) every loop round the wire gives the same circulation, $\mu_0 I$, whatever its shape or the wire's position, because the curl is zero away from the wire; a wire outside the loop gives zero, so the coil measures only the current it encloses