Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

$\Omega = 20\,000\tan(0.2\pi) = \mathbf{14\,531\ rad/s}$; $2\pi \times 2000 = 12\,566$: **16**% higher; the transform squeezes all analogue frequencies into 0 to $f_s/2$, more strongly near $f_s/2$

2.2

$s = 0 \to \mathbf{z = 1}$, DC; $s = \infty \to \mathbf{z = -1}$, $f_s/2$; the left half-plane maps inside the unit circle, so poles stay inside

2.3

$N \geq \dfrac{\log_{10}(10^4 - 1)}{2\log_{10} 2} = 6.64$; $N = \mathbf{7}$

2.4

DC, $z = 1$: $\dfrac{0.2 \times 2}{0.4} = \mathbf{1}$; $z = -1$: $\mathbf{0}$; pole at 0.6, inside the circle: **stable**

3.1

$\Omega_c = 2000\tan(0.1\pi) = \mathbf{649.8\ rad/s}$

$b_0 = \dfrac{649.8}{2649.8} = \mathbf{0.2452}$

$a_1 = \dfrac{649.8 - 2000}{2649.8} = \mathbf{-0.5095}$

$H(z) = 0.2452\dfrac{1 + z^{-1}}{1 - 0.5095z^{-1}}$

DC: $\dfrac{0.4905}{0.4905} = \mathbf{1}$

3.2

$|1 + e^{-j\omega}| = 2\cos(0.1\pi) = \mathbf{1.902}$

$|1 - 0.5095e^{-j\omega}| = \sqrt{1 - 1.019\cos 0.2\pi + 0.260} = \mathbf{0.660}$

$|H| = \dfrac{0.2452 \times 1.902}{0.660} = \mathbf{0.707}$, -3 dB as designed

without prewarping: $f = \dfrac{f_s}{\pi}\tan^{-1}\dfrac{628.3}{2000} = \mathbf{96.9\ Hz}$, 3% low

3.3

(a) $\Omega_c = 96\,000\tan\left(\dfrac{\pi \times 2000}{48\,000}\right) = \mathbf{12\,639\ rad/s}$; 0.6% above 12 566

(b) $b_0 = \dfrac{12\,639}{108\,639} = \mathbf{0.1163}$; $a_1 = \mathbf{-0.7673}$

(c) the zero at $z = -1$ makes the gain **0** at 24 kHz; warping squeezes the analogue response from 0 to ∞ into 0 to 24 kHz, so it falls faster near $f_s/2$

(d) yes: the error is 1.3 dB at 10 kHz, and above about 15 kHz a guitar has little to hear; to get closer, run the filter at a higher rate (96 kHz) so the warping moves out of the audio band

exercises.lol exercises.com.cn