

Solutions

2.1

- $\omega_a = 2000 \tan(\pi/4) = 2000 \text{ rad/s}$.

2.2

- Maps to a real z with $|z| < 1$: the whole left half-plane goes strictly inside the unit circle, so stable analog points stay stable digital points.

2.3

- Stable: both poles satisfy $|z| < 1$ – for rational systems BIBO stability is exactly all poles inside the unit circle.

3.1

- Prewarp: $\omega_a = 16\,000 \tan(\pi/8) = 6627 \text{ rad/s}$, $K = 1/\tan(\pi/8) = 2.414$.
- $s/\omega_a = K(1 - z^{-1})/(1 + z^{-1})$ in $H = 1/(s/\omega_a + 1)$: $H(z) = \frac{1+z^{-1}}{(K+1)+(1-K)z^{-1}}$.
- Numbers: $H(z) = \frac{1+z^{-1}}{3.414-1.414z^{-1}}$.
- DC gain $2/2 = 1.000$ ✓; at 1 kHz the prewarp places -3.01 dB exactly ✓.

3.2

- Lands at 250 Hz: the -3 dB point sits where $\tan(\omega T/2) = 1$, i.e. $f = f_s/4$.
- At 100 Hz the naive filter reads -0.05 dB – a complacent passband number that hides the miss.
- Near dc $\tan \theta \approx \theta$ so warp vanishes; near Nyquist the tangent compresses violently, so unwarped designs land far off.