Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

DFT $256^2 = \mathbf{65\,536}$; FFT $128 \times 8 = \mathbf{1024}$; ratio **64**

2.2

$\dfrac{8000}{512} = \mathbf{15.6\ Hz}$; bin 64: **1000 Hz**; bin $448 = 512 - 64$: $\mathbf{-1000\ Hz}$, the mirror of bin 64

2.3

$\log_2 8 = \mathbf{3}$ stages; $\dfrac{8}{2} = \mathbf{4}$ butterflies each; $3 \times 4 = \mathbf{12}$ complex multiplies

2.4

5120 complex multiplies; $4 \times 5120 = \mathbf{20\,480}$ real multiplies; adds: $2 \times 5120 + 2 \times 10\,240 = \mathbf{30\,720}$ real adds

3.1

frame $\dfrac{2048}{48\,000} = \mathbf{42.7\ ms}$

$4 \times 1024 \times 11 = \mathbf{45\,056}$ real multiplies

time $\dfrac{45\,056}{10^8} = \mathbf{0.45\ ms}$

load $\dfrac{0.45}{42.7} = \mathbf{1.1}\%$

with 50% overlap, twice the frames: $\mathbf{2.1}\%$

3.2

direct: $512 \times 48\,000 = \mathbf{24.6\ M}$ per second

per block: $2 \times 5120 + 1024 = 11\,264$ complex multiplies $= 45\,056$ real

per output sample $\dfrac{45\,056}{513} = 87.8$; per second $\mathbf{4.2\ M}$

the FFT route is about **5.8 times** cheaper

3.3

(a) $N \geq 2400$: $N = \mathbf{4096}$; bins $\dfrac{2.4 \times 10^6}{4096} = \mathbf{586\ Hz}$

(b) $\dfrac{4096}{2} \times 12 = 24\,576$ complex; $\mathbf{98\,304}$ real

(c) $\dfrac{2.4 \times 10^6}{4096} = 586$ FFTs per second, 57.6 M per second: load $\mathbf{2.9}\%$; about **23** FFTs per display

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(d) every sample: the load is small, and averaging 23 spectra smooths the noise; only 25 FFTs would watch 4% of the time and miss short signals; the cost: averaging also hides very short bursts