

Solutions

2.1

- $\log_2 64 = 6$ stages, each $N/2 = 32$ butterflies.

2.2

- $\Delta f = 8000/512 = 15.625$ Hz.
- Bin 90: $90 \times 15.625 = 1406.25$ Hz.

2.3

- Direct $256^2 = 65\,536$; FFT $256 \times 8 = 2048 - 32\times$ fewer.

3.1

- $\Delta f = 48\,000/1024 = 46.875$ Hz.
- FFT count $1024 \times 10 = 10\,240$ operations.
- At 5 ns: $51.2 \mu s$ per transform.
- Margin: $51.2 \mu s$ against 21.3 ms – about $400\times$ headroom ✓.
- 1 kHz: $1000/46.875 = 21.33 \rightarrow$ bin 21, exact frequency $21 \times 46.875 = 984.375$ Hz (0.984 kHz).

3.2

- Two 512-FFTs: $2 \times 512 \times 9 = 9216$, plus 512 butterflies = 9728 operations – versus 10 240: a 5% saving only.
- Danielson–Lanczos: one 1024 DFT IS two 512 DFTs plus N twiddle combines – the proposal reinvents the first FFT stage.
- Splitting helps pipelining: shorter transforms mean smaller, shallower butterfly arrays with cleaner memory locality.