

Solutions

2.1

- $N = f_s/\Delta f = 2000/4 = 500$.

2.2

- Amplitude 2 $\rightarrow$ each exponential carries amplitude 1, summing over N samples:
 $|X[3]| = |X[5]| = A N/2 = 2 \times 4 = 8$, all other bins 0.

2.3

- $\Delta f \leq 3$ Hz needs $N \geq 1000/3 = 334 \rightarrow N = 334$ or more (0.334 s).
- Padding only interpolates the same spectrum – no new information to separate the tones.

3.1

- $\Delta f < 5$ Hz separation needs $N > 1000/5 = 200$: minimum $N = 200$ (0.2 s).
- $N = 256$: $\Delta f = 3.906$ Hz.
- $3.906 < 5 \rightarrow$ resolvable $\checkmark$ (bins $50/3.906 = 12.8$ and $55/3.906 = 14.08$ – different bins).
- Padding to 2048 draws the same underlying spectrum with $8\times$ more points (prettier plot); it cannot create the frequency separation that the 0.2 s record already fixed.

3.2

- Sampling in frequency $\Rightarrow$ periodic time: a 1.5-bin tone's periodic copies do not align, so the mismatch smears into every bin – leakage.
- Largest: $|X[2]| = |X[6]| = 2.80$, then $|X[1]| = |X[7]| = 2.40$.
- Hamming: sidelobes drop from -13 dB to -43 dB (small tones beside large ones reappear).
- Main lobe widens (about $2\times$ the rectangle): two nearby tones now blur together – the trade is dynamic range against resolution.