

Solutions

2.1

- Diagonals: $y = 1, 4, 6, 5, 2$.

2.2

- $\sum 0.4^n = 1/(1 - 0.4) = 1.667$ converges ✓ stable.

2.3

- $H(1) = 2 - 1 + 2 = 3$; steady step output 3 ✓.

3.1

- $a = 2$: step response $1, 3, 4, 4, \dots$, settling at $\sum h = 4$.
- Non-negative everywhere needs $y[1] = 1 + a \geq 0$ and $y[3] = 2 + a \geq 0$: the binding sample is $n = 1$, so $a \geq -1$.
- Causal: $h[n] = 0$ for $n < 0$ ✓ (finite taps).
- BIBO: finite $\sum |h| = |1| + |a| + |1|$ always converges ✓ every FIR is unconditionally stable.

3.2

- Impulse response $h[n] = \alpha^n$, $n \geq 0$.
- Step: $1, 1.5, 1.75, 1.875$ ✓.
- Closed form $y[n] = (1 - \alpha^{n+1})/(1 - \alpha)$; at $\alpha = 0.5$: $2 - 2^{-n}$ ✓ (limit 2).
- Pole at $z = \alpha$; stable for $0 < \alpha < 1$ ($\sum \alpha^n$ converges).
- Within 1% of limit 2: remaining gap $2^{-n} \leq 0.02 \Rightarrow n \geq 6$ samples.