Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

$\dot{x}_1 = x_2$; $\dot{x}_2 = -3x_1 - 4x_2 + 2u$, so $A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$; $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

2.2

$\det(sI - A) = s^2 + 6s + 8$; $s = \mathbf{-2}$, $\mathbf{-4}$; both negative: **stable**

2.3

$(sI - A)^{-1} = \dfrac{1}{s^2 + 3s + 2} \begin{bmatrix} s + 3 & 1 \\ -2 & s \end{bmatrix}$; $C(\ldots)B$ picks the top-right entry; $G = \dfrac{1}{s^2 + 3s + 2}$

2.4

$AB = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$, so $[B \;\; AB] = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$; rank **1**: **not controllable**; nothing connects u to x_2, so its pole at -2 cannot be moved

3.1

$m\ddot{y} + b\dot{y} + ky = u$: $A = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

$s^2 + 2s + 5 = 0$: $s = \mathbf{-1 \pm j2}$

$\omega_n = \sqrt{5} = \mathbf{2.24}$ rad/s, $\zeta = \dfrac{2}{2\sqrt{5}} = \mathbf{0.45}$

$G = \dfrac{1}{s^2 + 2s + 5}$

3.2

$BK = \begin{bmatrix} 0 & 0 \\ 50k_1 & 50k_2 \end{bmatrix}$, $A - BK = \begin{bmatrix} 0 & 1 \\ -50k_1 & -35 - 50k_2 \end{bmatrix}$

$\det = s^2 + (35 + 50k_2)s + 50k_1$

$50k_1 = 400$: $k_1 = \mathbf{8}$ V/rad; $35 + 50k_2 = 28$: $k_2 = \mathbf{-0.14}$ V s/rad

$k_2 < 0$: the motor's own damping (35) is more than the 28 we want, so speed feedback must cut it; the design is faster and less damped than the bare motor

3.3

(a) $\det = s^2 + (2 + k_2)s + k_1$: $K = \begin{bmatrix} 25 & 5 \end{bmatrix}$; $u(0) = -25 \times 0.1 = \mathbf{-2.5}$

(b) $K = \begin{bmatrix} 400 & 26 \end{bmatrix}$; $u(0) = \mathbf{-40}$, four times the limit

(c) $0.1k_1 \le 10$: $k_1 = \omega_n^2 \le 100$, $\omega_n = \mathbf{10}$, $K = \begin{bmatrix} 100 & 12 \end{bmatrix}$; $t_s = \dfrac{4}{0.7 \times 10} = \mathbf{0.57}$ **s**

(d) no: the actuator would saturate, so the loop is no longer linear, the placed poles no longer apply, and the response overshoots or winds up; use $\omega_n = 10$, or a larger actuator if 0.57 s is too slow

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