Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

P: $K_p = 0.5 \times 20 = \mathbf{10}$; PI: $K_p = 0.45 \times 20 = \mathbf{9}$, $T_i = \dfrac{2}{1.2} = \mathbf{1.67\ s}$

2.2

$K_i = \dfrac{10}{2} = \mathbf{5}$; $K_d = 10 \times 0.5 = \mathbf{5}$; the $K_d s$ (derivative) term

2.3

K_p (position constant) $= 5 \times \dfrac{4}{2} = 10$; $e_{ss} = \dfrac{1}{1+10} = \mathbf{0.091}$; with $\dfrac{K_i}{s}$ the loop has an integrator, so the error goes to **0**: the integral keeps growing while any error is left

2.4

$\sin 50^\circ = 0.766 = \dfrac{1-\alpha}{1+\alpha}$; $\alpha = \mathbf{0.132}$; $\dfrac{1}{\sqrt{0.132}} = 2.75$, **8.8 dB**

3.1

$s^3 + 6s^2 + 8s + K$: $K_u = 6 \times 8 = \mathbf{48}$

$6\omega^2 = 48$, $\omega = 2.83$ rad/s, $T_u = \dfrac{2\pi}{2.83} = \mathbf{2.22\ s}$

$K_p = 0.6 \times 48 = \mathbf{28.8}$; $T_i = \mathbf{1.11\ s}$, $T_d = \mathbf{0.278\ s}$

$K_i = \dfrac{28.8}{1.11} = \mathbf{25.9}$, $K_d = 28.8 \times 0.278 = \mathbf{8.0}$

3.2

$\sin 37^\circ = 0.602$: $\alpha = \dfrac{0.398}{1.602} = \mathbf{0.249}$

the network adds $\dfrac{1}{\sqrt{\alpha}} = 2.0$ at ω_m, so ω_m goes where $|L| = \sqrt{0.249} = 0.499$: $\mathbf{4.42\ rad/s}$

$T = \dfrac{1}{4.42 \times 0.499} = \mathbf{0.453\ s}$

zero at $\dfrac{1}{T} = \mathbf{2.21}$, pole at $\dfrac{1}{\alpha T} = \mathbf{8.87}$ rad/s

plant phase at 4.42: $-90 - 77.3 = -167.3^\circ$; add 37°: $PM = \mathbf{49.7^\circ}$

3.3

(a) $28.8 \times 0.002 = \mathbf{0.058\ V}$

(b) $|K_d j\omega| = 8.0 \times 500 = 4000$; $4000 \times 0.002 = \mathbf{8.0\ V}$: a large buzz on the motor

(c) corner $\dfrac{N}{T_d} = \dfrac{10}{0.278} = \mathbf{36\ rad/s}$; above it the gain is flat at $K_d \times 36 = 288$: $288 \times 0.002 = \mathbf{0.58\ V}$

(d) 0.58 V is ten times the P term but 14 times less than before, so it is usually acceptable; the cost: the derivative now loses its phase lead above 36 rad/s, so it adds less damping near crossover, and a smaller N costs more

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