Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

$\dfrac{8}{s+4} = \dfrac{2}{0.25s+1}$: DC gain **2**; $\tau = \mathbf{0.25}$ **s**; $t_s = 4\tau = \mathbf{1}$ **s**

2.2

$M_p = e^{-0.4\pi/0.917} = \mathbf{25}\%$; $t_p = \dfrac{\pi}{10 \times 0.917} = \mathbf{0.34}$ **s**; $t_s = \dfrac{4}{4} = \mathbf{1.0}$ **s**

2.3

$\ln 0.1 = -2.303$; $\zeta = \dfrac{2.303}{\sqrt{\pi^2 + 2.303^2}} = \mathbf{0.59}$; $\omega_n = \dfrac{\pi}{0.5\sqrt{1 - 0.59^2}} = \mathbf{7.8}$ **rad/s**

2.4

A: $K_p = \dfrac{20}{10} = 2$, error $\dfrac{1}{3} = \mathbf{0.33}$; B: type 1, $K_v = \dfrac{20}{5} = 4$; ramp error $\mathbf{0.25}$

3.1

$\zeta = \dfrac{-\ln 0.25}{\sqrt{\pi^2 + \ln^2 0.25}} = \mathbf{0.40}$

$\omega_d = \dfrac{\pi}{0.6} = 5.24$ rad/s, so $\omega_n = \dfrac{5.24}{\sqrt{1 - 0.16}} = \mathbf{5.72}$ **rad/s**

poles $\mathbf{-2.31 \pm j5.24}$

$t_s = \dfrac{4}{2.31} = \mathbf{1.7}$ **s**; check that the response stays inside $\pm 2\%$ after 1.7 s and that the second peak is $0.25^2 \approx 6\%$ high, at 1.8 s

3.2

$\zeta \geq \dfrac{-\ln 0.05}{\sqrt{\pi^2 + \ln^2 0.05}} = \mathbf{0.69}$

$2\zeta\omega_n = 10$: $\omega_n = 7.25$; $K = \omega_n^2 = \mathbf{52.5}$

$t_s = \dfrac{4}{5} = \mathbf{0.8}$ **s**, fixed by the $10s$ term

$K_v = \dfrac{52.5}{10} = 5.25$: ramp error $\mathbf{0.19}$

3.3

(a) $2\zeta\omega_n = 8$ for every K, so $t_s = \dfrac{4}{\zeta\omega_n} = \mathbf{1}$ **s** > 0.5 s; K changes only ω_n, not the real part -4

(b) $K_v = \dfrac{K}{8} \geq 50$: $K \geq \mathbf{400}$; then $\omega_n = 20$, $\zeta = 0.2$ and $M_p = \mathbf{53}\%$

(c) $\zeta \geq 0.59$ and $\zeta\omega_n \geq 8$: for example $-8 \pm j10.9$, with $\omega_n \approx 13.5$

(d) a lead (or PD) controller to pull the poles left of -8, plus a lag (or PI) part, or a higher low-frequency gain, for the ramp error, because one gain cannot set both the pole position and K_v

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