

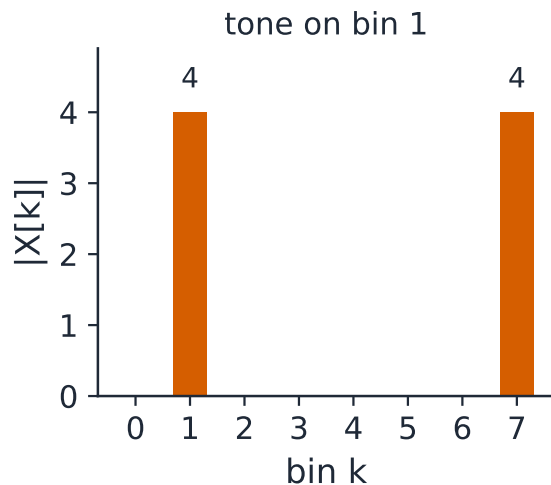
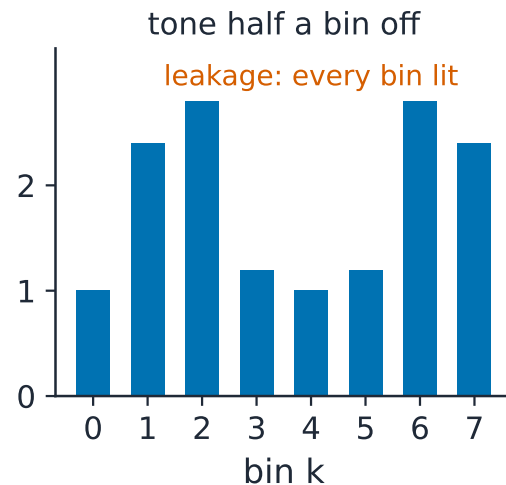
Exercise walk-through

12.6

Multirate processing

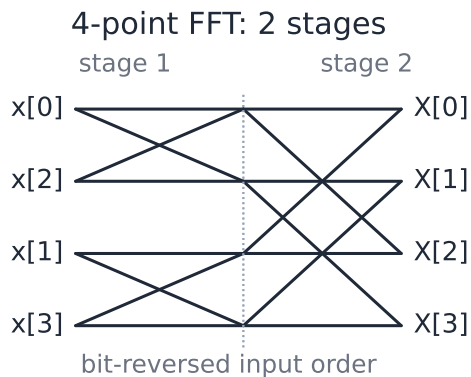
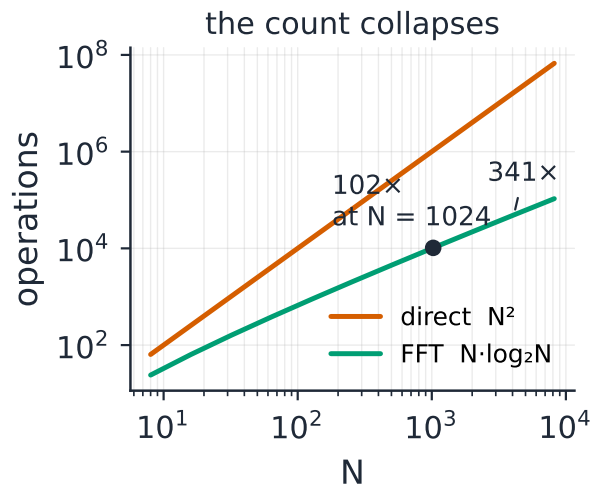
University Electrical Engineering

Two spikes become every bin



On bin 1: two clean spikes of 4. Half a bin off: every bin lit - **leakage** is the rectangular window speaking.

The butterfly ladder



Two stages of butterflies build a 4-point **FFT**; at $N = 1024$ the count collapses 102 $\times$ and keeps falling.

Multirate: work where it is cheap

- 1 **Up** 8 kHz speech to 48 kHz: interpolate by 6, low-pass at 4 kHz kills images at 8k, 16k, 40k; **polyphase** runs 6 subfilters at 8 kHz - 1/6 the multiplies.
- 2 **Ratio** 48 $\rightarrow$ 72 kHz is $L = 3$, $M = 2$: upsample to 144 kHz, ONE low-pass at 24 kHz stops the $\uparrow 3$ images and the $\downarrow 2$ aliases, keep every second sample.
- 3 **Gain** Zero-stuffing loses a factor L of amplitude - compensate it in the filter; forgetting the gain of L is the classic multirate bug.

Exercises 2.1-2.3

2.1 Decimate 48 kHz audio by $M = 2$: cutoff of the protective low-pass? [3]

2.2 Upsample 8 kHz speech by 3: at what frequencies do the first two images sit? [3]

2.3 Resample 44.1 kHz to 48 kHz – give a workable L and M . [3]

Exercises 2.1-2.3

2.1 Decimate 48 kHz audio by $M = 2$: cutoff of the protective low-pass? [3]

2.1

- $f_s/2M = 48/4 = 12$ kHz – everything above 12 kHz would fold.
-

2.2 Upsample 8 kHz speech by 3: at what frequencies do the first two images sit? [3]

2.2

- Images at multiples of 8 kHz around the original band: 8 kHz and 16 kHz (baseband copies centred there).
-

2.3 Resample 44.1 kHz to 48 kHz – give a workable L and M . [3]

2.3

- $48/44.1 = 160/147$: $L = 160$, $M = 147$ (the classic ratio).

Exercise 3.1

3.1 A 32 kHz system must deliver a 8 kHz output.

State the two stages and the order of filtering. Give the protective cutoff and the reason it cannot be skipped. Compute the output sample rate. Explain what changes if the low-pass is applied AFTER the decimation instead. [7]

Exercise 3.1

3.1 A 32 kHz system must deliver a 8 kHz output.

State the two stages and the order of filtering. Give the protective cutoff and the reason it cannot be skipped. Compute the output sample rate. Explain what changes if the low-pass is applied AFTER the decimation instead. [7]

3.1

- Stages: low-pass below $f_s/2M = 4$ kHz, then keep every 4th sample ($M = 4$).
- Output rate $32/4 = 8$ kHz ✓.
- Everything above 4 kHz (the new Nyquist) folds onto the kept band – aliasing is permanent, so the fence goes up first.
- Filtering after the drop cannot undo the fold: the aliases are already inside the band – it only removes what was already destroyed.

Exercise 3.2

3.2 An audio path upsamples 8 kHz speech to 48 kHz with a 200-tap low-pass.

Compute the multiplies per second running naively at 48 kHz. Compute the polyphase arrangement's rate (6 subfilters at 8 kHz) and its multiplies per second. State the gain factor missing when zero-stuffing (and where to put it), and explain why polyphase is exact, not approximate. [8]

Exercise 3.2

3.2 An audio path upsamples 8 kHz speech to 48 kHz with a 200-tap low-pass.

Compute the multiplies per second running naively at 48 kHz. Compute the polyphase arrangement's rate (6 subfilters at 8 kHz) and its multiplies per second. State the gain factor missing when zero-stuffing (and where to put it), and explain why polyphase is exact, not approximate. [8]

3.2

- Naive: $200 \times 48\,000 = 9.6$ M multiplies/s.
- Polyphase: $200/6 \approx 33$ effective taps per subfilter input at 8 kHz – total $200 \times 8\,000 = 1.6$ M/s ✓ one sixth.
- Zero-stuffing loses a factor $L = 6$ of amplitude: the filter needs a passband gain of 6 (or scale by L after).
- Polyphase is bookkeeping, not approximation: the same multiplies against the same coefficients, regrouped by which input samples can ever reach which output samples.