

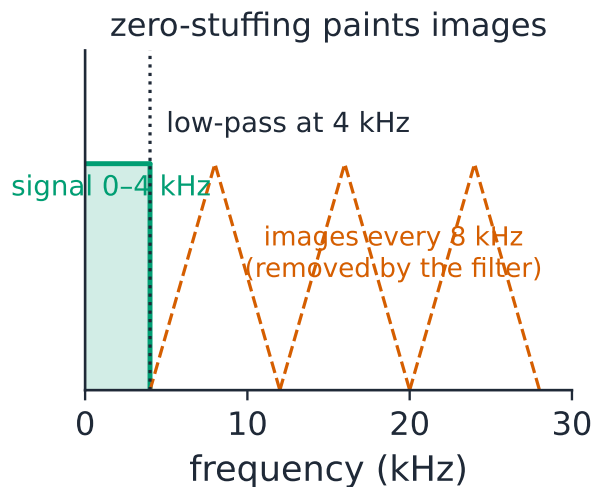
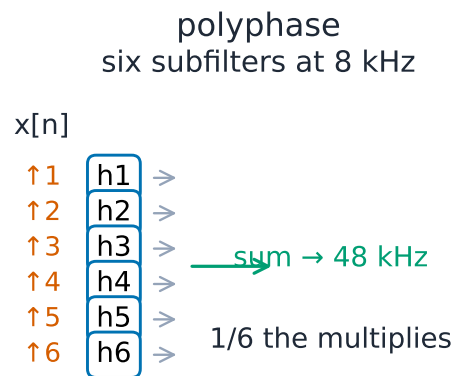
Exercise walk-through

12.5

IIR filter design

University Electrical Engineering

Images every 8 kHz



Stuffing 5 zeros paints images every 8 kHz; one low-pass at 4 kHz erases them;
polyphase pays 1/6 the multiplies.

The project: specify, verify, measure

- 1 **Spec** 1 Hz bins at $f_s = 1$ kHz needs $N \geq 1000$ - already 1.0 s against the <1 s latency line: the conflict surfaces at design time, exactly when you want it.
- 2 **Stages** Filter $\rightarrow$ window $\rightarrow$ **FFT** $\rightarrow$ threshold, each verified alone: a defect found in a stage costs an hour; in the integrated system, a week of bisection.
- 3 **Report** The final table lists spec, measured, margin per line - 5 of 6 and honest beats 6 of 6 claimed with 4 measured.

Four blocks and a margin table

verify each stage alone — then measure the whole



spec	measured	margin
1 Hz	0.977 Hz	ok
<1 s	1.024 s	✗
-10 dB	-10 + 27 dB	ok

Each block carries its own check; the pipeline ends in the spec-versus-measured margin table with no silent goals.

Exercises 2.1-2.3

2.1 Prewarp a 250 Hz cutoff at $f_s = 1$ kHz. [3]

2.2 What does the bilinear transform do to the analog point $s = \sigma + j0$ with $\sigma < 0$? [3]

2.3 A digital filter has poles at $z = 0.9$ and $z = 0.5$. Stable? Why? [3]

Exercises 2.1-2.3

2.1 Prewarp a 250 Hz cutoff at $f_s = 1$ kHz. [3]

2.1

- $\omega_a = 2000 \tan(\pi/4) = 2000$ rad/s.

2.2 What does the bilinear transform do to the analog point $s = \sigma + j0$ with $\sigma < 0$? [3]

2.2

- Maps to a real z with $|z| < 1$: the whole left half-plane goes strictly inside the unit circle, so stable analog points stay stable digital points.

2.3 A digital filter has poles at $z = 0.9$ and $z = 0.5$. Stable? Why? [3]

2.3

- Stable: both poles satisfy $|z| < 1$ – for rational systems BIBO stability is exactly all poles inside the unit circle.

Exercise 3.1

3.1 Design a 1st-order Butterworth low-pass at 1 kHz cutoff, $f_s = 8$ kHz, by bilinear transform.

Prewarp the cutoff. Apply the transform with the prewarped constant to $H(s) = 1/(s+1)$ (s normalised by ω_a), giving $H(z)$ as $b_0(1 + z^{-1})/(1 + a_1z^{-1})$. Verify the dc gain and state the response at the cutoff frequency. [8]

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3.1

- Prewarp: $\omega_a = 16\,000 \tan(\pi/8) = 6627$ rad/s, $K = 1/\tan(\pi/8) = 2.414$.
- $s/\omega_a = K(1-z^{-1})/(1+z^{-1})$ in $H = 1/(s/\omega_a + 1)$: $H(z) = \frac{1+z^{-1}}{(K+1)+(1-K)z^{-1}}$.
- Numbers: $H(z) = \frac{1+z^{-1}}{3.414-1.414z^{-1}}$.
- DC gain $2/2 = 1.000$ ✓; at 1 kHz the prewarp places -3.01 dB exactly ✓.

Exercise 3.2

3.2 The naive substitution ($K = 1$, no prewarp) is used for a 100 Hz, $f_s = 1$ kHz 2nd-order Butterworth.

State where the -3 dB point lands and why. Compute the response the naive filter shows at 100 Hz. Explain in one sentence why the error is small at low cutoffs and fatal near Nyquist. [7]

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3.2

- Lands at 250 Hz: the -3 dB point sits where $\tan(\omega T/2) = 1$, i.e. $f = f_s/4$.
- At 100 Hz the naive filter reads -0.05 dB – a complacent passband number that hides the miss.
- Near dc $\tan \theta \approx \theta$ so warp vanishes; near Nyquist the tangent compresses violently, so unwarped designs land far off.