Exercise walk-through

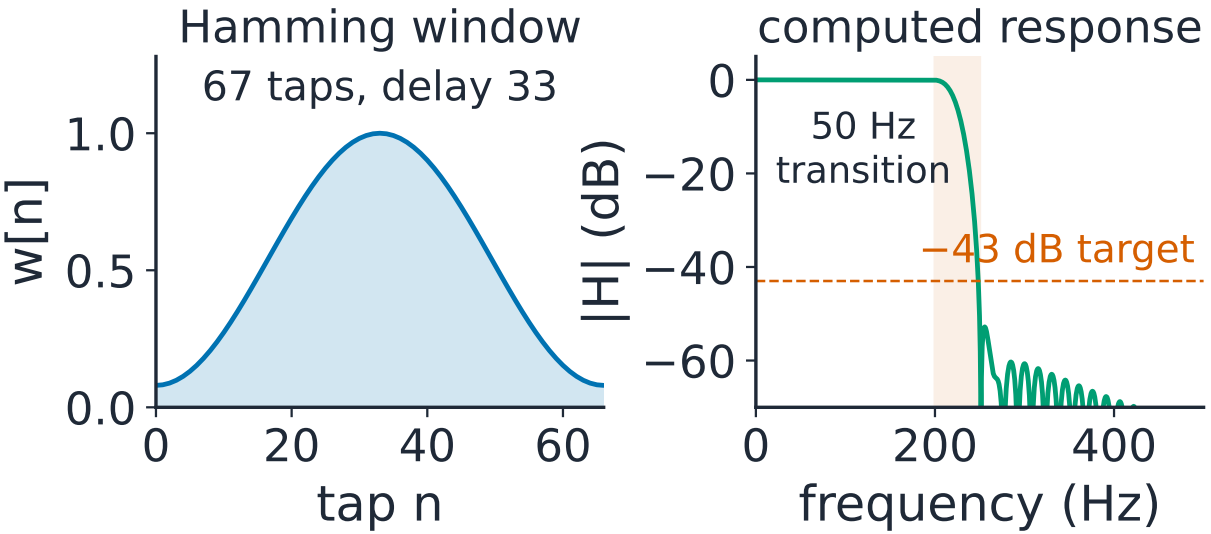
12.4
FIR filter design

University Electrical Engineering

FIR: the window is the fate

1. **Size** Hamming at $f_s = 1$ kHz, 50 Hz transition: $N = 3.3 \cdot 1000/50 = 66$ - take 67 (odd, Type-I), delay 33 samples; $N = 31$ gives 106 Hz: these are transition-width estimates.
2. **Verify** Measure the response from the taps: target stopband at or below -43 dB and ripple inside ± 0.05 dB. A -37 dB result fails: inspect taps, cutoff, window, N and measurement band.
3. **Optimal** Windows are convenient, not optimal - the alternation theorem behind Parks-McClellan makes equiripple the optimum.

Window in, response out



67 Hamming taps, 225 Hz ideal cutoff: computed stopband from 250 Hz peaks near −52.3 dB; delay 33 samples.

Exercises 2.1·2.3

2.1 Hamming FIR, $f_s = 8$ kHz, transition 100 Hz: estimate N and the linear-phase delay for the odd choice. [3]

2.2 Which of Hann, Hamming and Blackman has a tabulated first window sidelobe below -50 dB? Explain why this comparison does not verify a designed filter's stopband. [3]

2.3 Why must a Type-I design use an odd N? [3]

Exercises 2.1·2.3

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2.1

- $N = 3.3 \times 8000/100 = 264 \rightarrow 265$ taps, delay 132 samples (16.5 ms).

2.2 Which of Hann, Hamming and Blackman has a tabulated first window sidelobe below -50 dB? Explain why this comparison does not verify a designed filter's stopband. [3]

2.2

- Blackman (-58 dB) is below -50 dB; the tabulated Hann and Hamming window sidelobes are not. These are window-spectrum levels, so the filter still needs its own frequency-response check.

2.3 Why must a Type-I design use an odd N? [3]

2.3

- Type-I symmetry needs a centre tap: $h[n] = h[N-1-n]$ has an exact middle only when N is odd, giving whole-sample delay $(N-1)/2$.

Exercise 3.1

3.1 Design a Hamming low-pass at $f_s = 1$ kHz: passband to 200 Hz, transition 50 Hz. The design targets are stopband level at or below -43 dB and passband ripple within ± 0.05 dB.

Size N (odd). State the delay in samples and milliseconds. Compute what transition $N = 31$ would deliver, and state which way the design misses. Give the two measurements that verify the design, with their expected values. [8]

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3.1

- $N = 3.3 \times 1000/50 = 66 \rightarrow 67$ taps.
- Delay $(67 - 1)/2 = 33$ samples $= 33$ ms.
- $N = 31$: transition $3.3 \times 1000/31 = 106$ Hz – too wide (the design misses on sharpness).
- Verify: stopband at or below -43 dB and passband ripple $\leq \pm 0.05$ dB, both computed from the taps.

Exercise 3.2

3.2 An anti-alias FIR before a $\downarrow 4$ decimator at $f_s = 48$ kHz must pass 0–5 kHz and attenuate above 6 kHz.

Compute the transition band available and the required N (Hamming). The group delay of this filter in ms; the decimator drops 3 of 4 samples – explain why the filter must run BEFORE the drop. Compare the multiply rate of filtering at 48 kHz versus a polyphase scheme. [8]

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3.2

- Transition $5 \rightarrow 6$ kHz $= 1$ kHz wide.
- $N = 3.3 \times 48\,000/1000 = 158.4 \rightarrow 159$ taps.
- Delay $(159 - 1)/2 = 79$ samples at 48 kHz $= 1.65$ ms.
- Dropping first folds everything above 6 kHz into the kept band – aliasing is permanent; the fence goes up before the drop.
- At 48 kHz: $159 \times 48\,000 = 7.6$ M multiplies/s; polyphase runs the same taps at 12 kHz: 1.9 M/s – one quarter.