

Electrical Machines and Power Systems

University Electrical Engineering

What this course is, and how to use it

This is the ninth course in University Electrical Engineering: **electrical machines and power systems**. EE07 wrote the field equations; this course puts them to work – the machines that convert mechanical and electrical energy, and the networks that move it.

- Unit 9.1 builds the magnetic circuit and the **transformer**.
- Units 9.2–9.4 cover the three machine families: DC, induction, synchronous.
- Units 9.5–9.6 widen to the network: per-unit arithmetic and power flow.
- Unit 9.7 opens power electronics.

The running story is one small workshop: a 10 kVA transformer feeds a distribution board; an induction motor drives a compressor; a synchronous generator stands behind the grid; a buck converter charges the battery backup. Every machine answer below is checked through a second route – loss accounting, the equivalent circuit at a limit, or the power balance – and every per-unit answer by converting back to real units.

Magnetic circuits and transformers

Why it matters. The charger that fills your phone in an hour contains a transformer smaller than a coin, and it works because magnetic flux, like current, flows in loops through materials that resist it. The same picture, scaled up a million times, moves power between continents.

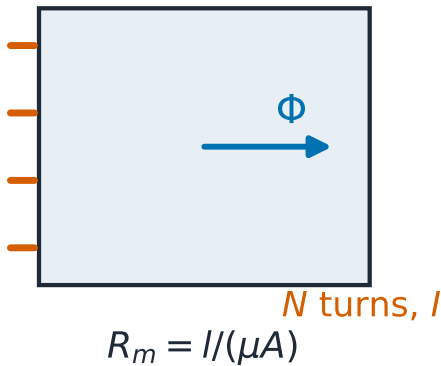
The idea, with a picture. A coil of N turns carrying current I is a pump for **flux**: it creates **magnetomotive force** $F = NI$ (ampere-turns). Flux Φ (weber) flows around an iron loop of length l and cross-section A , opposed by **reluctance** $R_m = \frac{l}{\mu A}$, where $\mu = \mu_0 \mu_r$ is the permeability. This is the magnetic mirror of Ohm's law:

$$\Phi = \frac{F}{R_m}, \quad \text{just as} \quad I = \frac{V}{R}.$$

The flux density $B = \Phi/A$ (tesla) is the quantity iron cares about – power-frequency cores work between about 1.0 and 1.5 T.

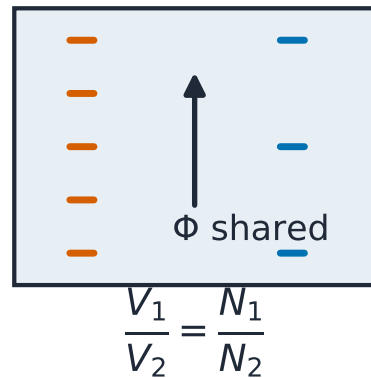
(1) the magnetic circuit

$F = NI$ drives $\Phi = F/R_m$



(2) the transformer

V_1, N_1 turns V_2, N_2 turns



One flux loop, two coils: the transformer is just a magnetic circuit shared by two windings

The derivation. Ampère's law around the core gives $Hl = NI$, with H the field intensity in A/m. With $B = \mu H$ and $\Phi = BA$:

$$\Phi = \mu A \cdot \frac{NI}{l} = \frac{NI}{l/(\mu A)} = \frac{F}{R_m}.$$

Nothing deeper – the whole unit is bookkeeping on this line, plus one warning: air has $\mu_r = 1$, four thousand times weaker than the core below, so a tiny air gap can dominate the reluctance of the entire loop.

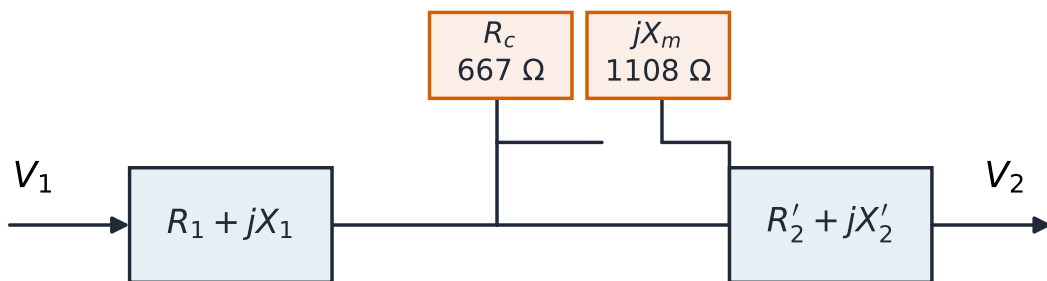
Worked example 1 – a core on its own. A core of mean length 0.5 m, cross-section $10 \text{ cm}^2 = 10^{-3} \text{ m}^2$, $\mu_r = 4000$, wound with 200 turns at 0.5 A. The magnetomotive force is $F = 200 \times 0.5 = 100 \text{ At}$. The reluctance is $R_m = \frac{0.5}{4\pi \times 10^{-7} \times 4000 \times 10^{-3}} = 99\,472 \text{ At/Wb}$, so the flux is $\Phi = \frac{100}{99\,472} = 1.005 \text{ mWb}$ and the density $B = \frac{1.005 \times 10^{-3}}{10^{-3}} = 1.005 \text{ T}$. Second route: $H = NI/l = 200 \text{ A/m}$, so $B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times 4000 \times 200 = 1.005 \text{ T}$ ✓ – a sensible power-frequency density, which is itself the sanity check that the arithmetic stayed in range.

Now cut a 0.5 mm gap in the same core. The gap adds $R_{\text{gap}} = \frac{0.5 \times 10^{-3}}{4\pi \times 10^{-7} \times 10^{-3}} = 397\,887 \text{ At/Wb}$ – four times the whole core's reluctance. Flux collapses to $\frac{100}{497\,359} = 0.201 \text{ mWb}$, one fifth, from a gap a hair thick. This is why transformer cores are stacked and laminated tight, and why gapped cores must be designed on purpose.

Worked example 2 – the transformer, measured. A transformer links two windings through one shared flux, so ideally $\frac{V_1}{V_2} = \frac{N_1}{N_2}$ and $\frac{I_1}{I_2} = \frac{N_2}{N_1}$: volts trade against amps, power stays. Real ones lose a little in the winding resistance (**copper loss** I^2R) and in the iron (**core loss** : hysteresis plus **eddy currents**). The two losses are separated by two tests:

- **Open-circuit test** (LV side, rated voltage): 400 V draws 0.7 A and 240 W. The 240 W is core loss. Its loss current is $I_c = 240/400 = 0.6$ A, so the magnetising branch is $R_c = 400/0.6 = 667 \Omega$ and, from $I_m = \sqrt{0.7^2 - 0.6^2} = 0.361$ A, $X_m = 400/0.361 = 1108 \Omega$ (both referred to the LV side).
- **Short-circuit test** (HV side, reduced voltage, rated current): 80 V draws the rated 5 A ($= 10\,000/2000$) and 300 W. That 300 W is the full-load copper loss, and the series branch is $R_{eq} = 300/5^2 = 12 \Omega$, $Z_{eq} = 80/5 = 16 \Omega$, $X_{eq} = \sqrt{16^2 - 12^2} = 10.6 \Omega$ (HV side).

open-circuit test fills these in



short-circuit test: $R_{eq} = 12 \Omega$, $Z_{eq} = 16 \Omega$, $X_{eq} = 10.6 \Omega$ (HV side)

Two tests, two branches: the open circuit fills in the shunt pair, the short circuit the series pair

Full-load efficiency at unity power factor: $\eta = \frac{10\,000}{10\,000 + 240 + 300} = 94.9\%$. At 0.8 power factor the output is 8000 W with the same losses, so $\eta = \frac{8000}{8540} = 93.7\%$ – fixed core loss hurts more as the useful power falls. Efficiency peaks where the load-dependent copper loss equals the fixed core loss: $x = \sqrt{240/300} = 0.894$ of full load, a number the workshop should recognise on its meter.

Common mistakes. Forgetting μ_0 inside μ (a factor of nearly a million, since $1/\mu_0 \approx 8 \times 10^5$). Referring OC values to the HV side or SC values to the LV side without squaring the turns ratio. Expecting the ideal-transformer current ratio to survive heavy loading – the series drop $I \times Z_{eq}$ is exactly what bends it. And treating an air

gap as a small detail: the example above shows a hair-thick gap is four fifths of the story.

Summary. $F = NI$ drives $\Phi = F/R_m$ through reluctance $l/(\mu A)$; density $B = \Phi/A$ must stay under about 1.5 T. Two tests on a real transformer separate shunt (core, from the open circuit) from series (copper, from the short circuit), and efficiency balances output against both.

DC machines

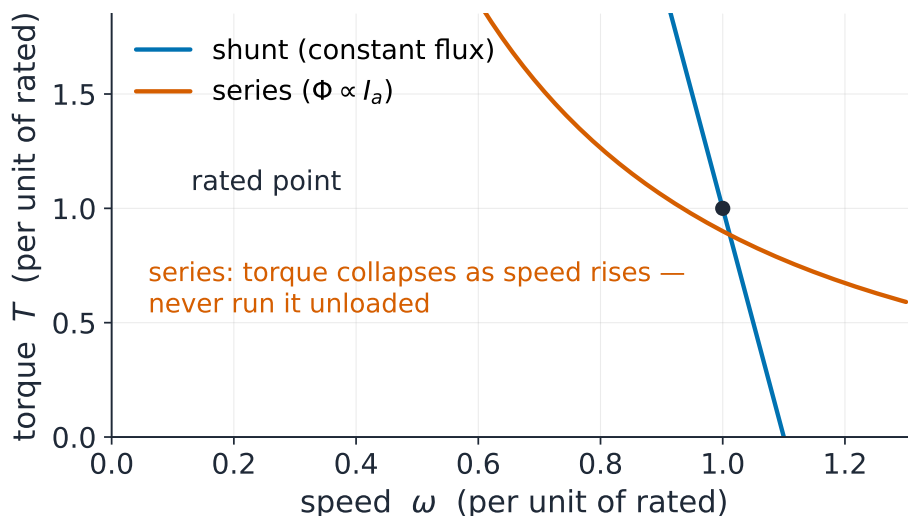
Why it matters. The motor in a cordless drill, an electric train's axle motor and the starter in every car with a combustion engine are DC machines: torque you can command with a current, from two lines of physics.

The idea, with a picture. A conductor of length l in field B carrying current I feels force $F = BIl$; the same conductor moving at speed v induces EMF $e = Blv$. A DC machine wires many such conductors around a rotor (the **armature**) and commutates them so each sees a fixed field direction – the **commutator** is a mechanical switch that keeps the torque unidirectional. Collect the bookkeeping into one constant $k\Phi$ and the whole machine is two equations:

$$E = k\Phi\omega \quad (\text{EMF}), \quad T = k\Phi I_a \quad (\text{torque}),$$

with $k\Phi$ set by the field flux alone. Multiplying them, $T\omega = EI_a$: the same air-gap power seen from both sides. Mechanical and electrical language are one language.

The field connection chooses the personality. A **shunt motor** (field in parallel, constant flux) holds nearly constant speed; a **series motor** (field in series, flux $\propto I_a$) makes torque $\propto I_a^2$ and must never run unloaded.



Shunt: a flat speed for any load. Series: huge torque at low speed, runaway risk at no load

The derivation. From $e = Blv$ with $v = \omega r$ and the geometry bundled into k : $E = k\Phi\omega$. From $F = BIl$ with the lever arms bundled the same way: $T = k\Phi I_a$. In a shunt machine Φ is fixed by the supply, so speed follows $\omega = \frac{V - I_a R_a}{k\Phi}$: the armature resistance drop is the only sag. In a series machine $\Phi \propto I_a$ (until iron saturates), so $T \propto I_a^2$ and $\omega \propto \frac{V - I_a R_a}{I_a}$ – speed rises sharply as load disappears.

Worked example 1 – the rating point. A shunt motor on 220 V with armature resistance $R_a = 0.2 \, \Omega$ draws $I_a = 50 \, \text{A}$ at 1500 rpm. The EMF is $E = 220 - 50 \times 0.2 = 210 \, \text{V}$. At 1500 rpm = 157.1 rad/s, $k\Phi = \frac{210}{157.1} = 1.337 \, \text{V} \cdot \text{s}$, so torque $T = 1.337 \times 50 = 66.9 \, \text{N} \cdot \text{m}$. Check both routes through the air gap: $T\omega = 66.9 \times 157.1 = 10.5 \, \text{kW}$ and $E I_a = 210 \times 50 = 10.5 \, \text{kW}$ ✓ – they must agree; they are the same conversion.

Starting the same motor from rest is the trap: at switch-on $E = 0$, so the inrush would be $I_a = \frac{220}{0.2} = 1100 \, \text{A}$, twenty times rated. A starter resistance limiting current to 100 A needs $R_{start} = \frac{220}{100} - 0.2 = 2.0 \, \Omega$, cut out in steps as E builds.

Worked example 2 – field weakening. Keep the load torque of 66.9 N · m and cut the field flux 20%, to $k\Phi = 1.070$. Torque demands $I_a = \frac{66.9}{1.070} = 62.5 \, \text{A}$, so the EMF settles at $E = 220 - 62.5 \times 0.2 = 207.5 \, \text{V}$ and the speed at $\omega = \frac{207.5}{1.070} = 193.9 \, \text{rad/s} = 1852 \, \text{rpm}$ – about 23% faster for one fifth less flux. Power check: $T\omega = 66.9 \times 193.9 = 12.96 \, \text{kW}$ against $E I_a = 207.5 \times 62.5 = 12.97 \, \text{kW}$ ✓. Above base speed this is the standard trick – **field weakening** : trade flux for speed, pay in armature current.

Common mistakes. Feeding rpm into $E = k\Phi\omega$ (it wants rad/s – a factor of 2π). Expecting a shunt motor to speed up under load (the drop across R_a makes it slow, slightly). Running a series motor unloaded – with no current there is no flux to limit the speed. And forgetting that at standstill the only thing limiting current is resistance.

Summary. $E = k\Phi\omega$ and $T = k\Phi I_a$, joined by $T\omega = E I_a$. Shunt wiring holds speed; series wiring multiplies torque at low speed. Start through resistance, weaken field for range.

Induction machines

Why it matters. Around seven of every ten industrial motors are **squirrel-cage** induction machines: no brushes, no commutator, one moving part – because the rotating field does the switching for free.

The idea, with a picture. Three coils spaced 120° apart, fed by three currents

spaced 120° apart in time, add to a **rotating field** of constant magnitude that rotates at **synchronous speed** $n_s = \frac{120f}{p}$ rpm (f in Hz, p poles). The rotor chases it and never quite catches up; the shortfall is **slip**

$$s = \frac{n_s - n}{n_s}.$$

Slip is the gear ratio of the machine: the rotor bars see the field slip past at rate s , so the rotor frequency is sf and every rotor quantity follows from it.

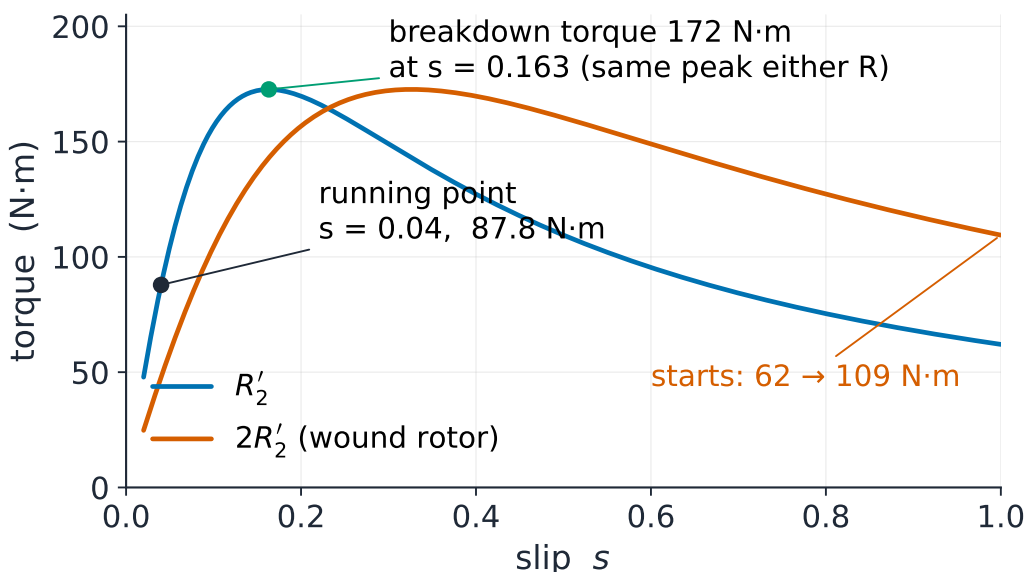
The derivation. In the equivalent circuit, all quantities are referred to the stator and the rotor branch appears as R'_2/s – the slip-scaled resistance. Split it:

$$\frac{R'_2}{s} = R'_2 + R'_2 \frac{1-s}{s},$$

and the second term is not a resistance at all: multiplied by I^2 it is exactly the mechanical output power, while $I^2 R'_2$ is the rotor copper loss. Slip divides the air-gap power $P_{ag} = T\omega_s$ into copper loss sP_{ag} and shaft power $(1-s)P_{ag}$. With a Thevenin source V_{th} , $Z_{th} = R_{th} + jX_{th}$ behind the magnetising branch, torque is

$$T = \frac{3V_{th}^2 (R'_2/s)}{\omega_s [(R_{th} + R'_2/s)^2 + (X_{th} + X'_2)^2]},$$

peaking (differentiate, set to zero) where $R'_2/s_{max} = \sqrt{R_{th}^2 + (X_{th} + X'_2)^2}$ – the **break-down torque**. Extra rotor resistance moves the peak towards starting: the **wound rotor** machine and the double-cage rotor sell exactly this.



Same peak torque, different address: rotor resistance chooses where the curve peaks

Worked example 1 – reading the nameplate. A 4-pole, 50 Hz motor: $n_s = \frac{120 \times 50}{4} = 1500$ rpm. Running at 1440 rpm: $s = \frac{60}{1500} = 0.04$. The rotor frequency is $sf = 2$ Hz – why rotor iron needs only light laminations – and the slip term R'_2/s is $25 \times R'_2$: at small slip the rotor behaves almost purely resistively, which is why the machine runs efficiently near synchronism.

Worked example 2 – the whole torque chain. The same motor, 400 V line (231 V per phase), $R_1 = 0.5$, $X_1 = 1.2$, $R'_2 = 0.4$, $X'_2 = 1.2 \Omega$, magnetising branch large (taken out of the way: $V_{th} = 231$ V, $Z_{th} = R_1 + jX_1$). Synchronous speed $\omega_s = 157.1$ rad/s. At $s = 0.04$, $R'_2/s = 10 \Omega$:

$$Z = 10.5 + j2.4, \quad |Z|^2 = 116.0, \quad T = \frac{3 \times 231^2 \times 10}{157.1 \times 116.0} = 87.8 \text{ N m}.$$

Follow the power: $P_{ag} = T\omega_s = 13.8$ kW, rotor copper loss $sP_{ag} = 552$ W, shaft power $T\omega_s(1 - s) = 13.2$ kW ✓ – one chain of multiplications, three balances closed. The peak: $s_{max} = \frac{0.4}{\sqrt{0.5^2 + 2.4^2}} = 0.163$, and the breakdown torque there is $172 \text{ N} \cdot \text{m}$ – twice the running point, the margin the compressor start needs. Locked rotor ($s = 1$): $T_{start} = 62 \text{ N} \cdot \text{m}$. Double the rotor resistance and T_{start} rises to $109 \text{ N} \cdot \text{m}$ while the running point (same slip) loses efficiency – the wound-rotor trade in one number.

Common mistakes. Mixing rpm and rad/s inside the torque formula (ω_s is mechanical, rad/s). Assuming torque rises with slip – it peaks at s_{max} and falls both sides. Charging the whole air-gap power to copper loss: copper gets only the fraction s . And expecting the rotor frequency to be 50 Hz – it is sf , 2 Hz here.

Summary. $n_s = 120f/p$, slip $s = (n_s - n)/n_s$, rotor frequency sf , rotor branch R'_2/s whose second half is the shaft. Torque peaks where R'_2/s matches the magnitude of $R_{th} + j(X_{th} + X'_2)$; rotor resistance buys starting torque at running cost.

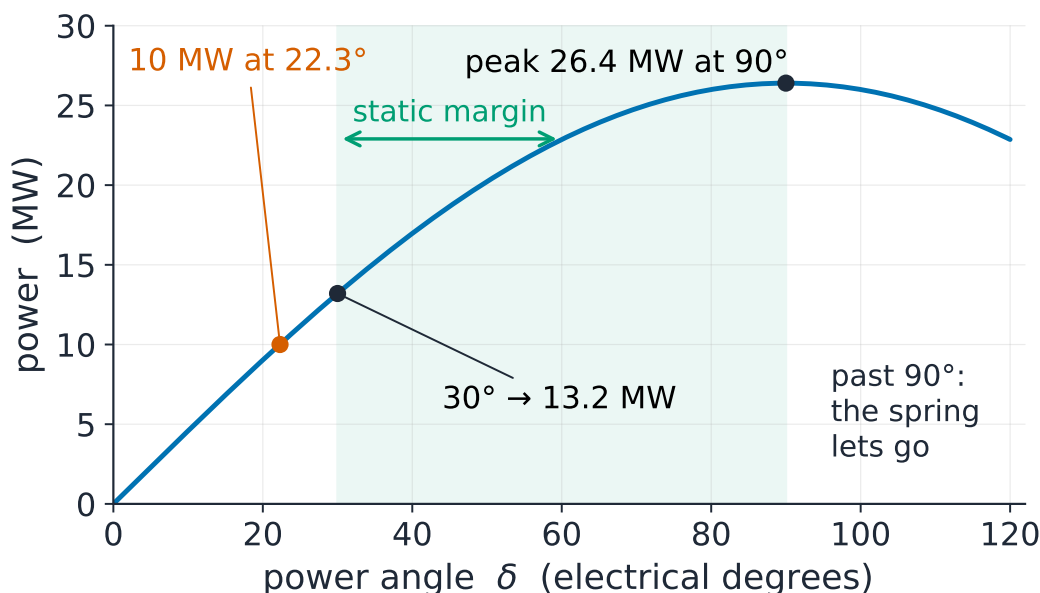
Synchronous machines

Why it matters. Virtually every watt from a power station flows out of a **synchronous machine** : a magnet spun by a turbine, locked to the grid's rotating field, delivering power through an angle.

The idea, with a picture. The rotor – an electromagnet or permanent magnet – locks to the rotating field, so speed is exactly $n_s = 120f/p$, no slip. The induced EMF has the transformer form $E = 4.44 f N \Phi k_w$ (the winding factor k_w tidies the coil geometry). What the machine exchanges with the grid is set by the **power angle**

δ : the angle by which the rotor's EMF leads the bus voltage. The connection behaves like a torsion spring – deflect it, feel torque:

$$P = \frac{3VE}{X_s} \sin \delta.$$



A sine-shaped spring: power grows with angle to 90 degrees, then lets go – that is loss of synchronism

The derivation. Per phase, the bus is $V\angle 0$, the internal EMF $E\angle\delta$, between them only the synchronous reactance X_s :

$$I = \frac{E\angle\delta - V}{jX_s}, \quad P = 3 \operatorname{Re}(VI^*) = \frac{3VE}{X_s} \sin \delta, \quad Q = \frac{3V(E \cos \delta - V)}{X_s}.$$

The Q line carries the second lesson: raising the field (**excitation**) raises E and pushes Q positive – the machine exports reactive power like a capacitor; lowering it imports. Grid operators schedule this daily.

Worked example 1 – the rating and its margin. A generator on an **infinite bus** at 11 kV line, $X_s = 5 \Omega$ per phase, EMF 12 kV line. Per phase $V = 6351$ V, $E = 6928$ V, so

$$P = \frac{3 \times 6351 \times 6928}{5} \sin \delta = 26.4 \sin \delta \text{ MW}.$$

At $\delta = 30^\circ$: $P = 13.2$ MW. The **static margin** is $\frac{26.4 - 13.2}{13.2} = 100\%$ – the machine could double its load before the spring lets go. Operators keep real machines nearer 30° for exactly this cushion.

Worked example 2 – real and reactive together. Same machine, schedule 10 MW: $\sin \delta = \frac{10}{26.4} = 0.379$, $\delta = 22.3^\circ$. Then

$$Q = \frac{3 \times 6351 \times (6928 \cos 22.3^\circ - 6351)}{5} = 0.23 \text{ MVar}.$$

Over-excited, it quietly supports the bus voltage while delivering the megawatts. Under-excite to $E = 10$ kV line (5774 V per phase) and hold 10 MW: $\sin \delta = \frac{10 \times 5 \times 10^6}{3 \times 6351 \times 5774} = 0.455$, $\delta = 27.0^\circ$, and $Q = \frac{3 \times 6351 \times (5774 \times 0.890 - 6351)}{5} = -4.6$ MVar: the same megawatts, but the machine now absorbs reactive power. One dial (excitation) moves Q far more than P .

Common mistakes. Using line voltages per phase (divide by $\sqrt{3}$ first). Believing more excitation gives more real power – the turbine sets P ; excitation sets Q . Pushing δ past 90° , where $\frac{dP}{d\delta}$ goes negative and the machine slips a pole. And forgetting that δ here is electrical: with p poles the mechanical angle is $2\delta/p$.

Summary. Speed locked to the grid, power $P = \frac{3VE}{X_s} \sin \delta$, reactive $Q = \frac{3V(E \cos \delta - V)}{X_s}$. Turbine owns the angle, field owns the VARs.

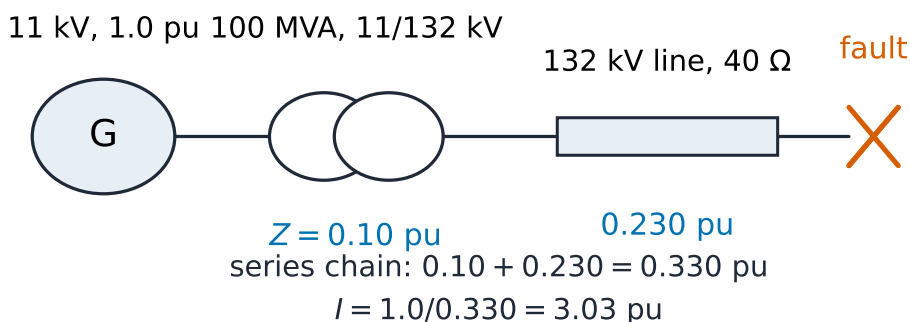
Power networks and the per-unit system

Why it matters. A grid calculation mixing ohms at 11 kV with ohms at 132 kV drowns in turns ratios. Power engineers long ago switched to a language where every voltage is near 1.0 and every impedance is a percentage: the **per-unit** system.

The idea, with a picture. Choose a **base value** pair: base power S_{base} and base voltage V_{base} at each level; everything else follows:

$$Z_{base} = \frac{V_{base}^2}{S_{base}}, \quad I_{base} = \frac{S_{base}}{\sqrt{3} V_{base}}, \quad Z_{pu} = \frac{Z_{actual}}{Z_{base}}.$$

Impedances move between voltage levels by the turns ratio squared – per-unit absorbs that automatically, because each level has its own Z_{base} . A transformer's nameplate "10%" means: at its own rated current, 10% of rated voltage drops inside it – which is exactly $Z_{pu} = 0.10$.



One line, one chain of per-unit impedances: the fault sees them all in series

The derivation. Base changes are pure bookkeeping. An impedance given as Z_{old} pu on S_{old} , V_{old} becomes, on new bases,

$$Z_{new} = Z_{old} \times \frac{S_{new}}{S_{old}} \times \left(\frac{V_{old}}{V_{new}} \right)^2,$$

and if the voltage bases already match the transformer ratio (as they do in a consistent study), only the S ratio survives.

Worked example 1 – a fault at the end of the line. A 100 MVA, 11/132 kV transformer with $Z = 10\%$ on its own base feeds a 132 kV line of 40Ω ; a three-phase fault closes at the line's far end with the generator held at 1.0 pu. Line base: $Z_{base} = \frac{132^2}{100} = 174.2 \Omega$, so the line is $\frac{40}{174.2} = 0.230$ pu; the transformer is already 0.10 pu on 100 MVA. Total 0.330 pu, so the **fault current** is $\frac{1.0}{0.330} = 3.03$ pu. Back to real units – the check that proves the method: base current at 132 kV is $I_{base} = \frac{100 \times 10^6}{\sqrt{3} \times 132\,000} = 437$ A, so $I_{fault} = 3.03 \times 437 = 1327$ A. The same 3.03 pu read on the 11 kV side with that level's base current $\frac{100 \times 10^6}{\sqrt{3} \times 11\,000} = 5249$ A gives $3.03 \times 5249 = 15\,926$ A – one dimensionless number serves both voltage levels ✓.

Worked example 2 – changing bases. The study grows: add a 60 MVA, 66/11 kV transformer rated 8% and a generator with $X_d = 0.25$ pu on 80 MVA, 11 kV. On the study's 100 MVA base the transformer becomes $0.08 \times \frac{100}{60} = 0.133$ pu and the generator $0.25 \times \frac{100}{80} = 0.313$ pu (voltage bases already agree). In series between the generator's EMF and a fault on the transformer's 66 kV terminals: $0.313 + 0.133 = 0.446$ pu, so the fault current is $\frac{1.0}{0.446} = 2.24$ pu. On the 66 kV side the base current is $\frac{100 \times 10^6}{\sqrt{3} \times 66\,000} = 875$ A, so $2.24 \times 875 = 1962$ A flows into the fault, while the generator carries $2.24 \times 5249 = 11\,780$ A on its 11 kV side – the same pu, two real currents. Two multiplications replaced a page of ratio chasing.

Common mistakes. Using line-to-line volts in a per-phase base (or vice versa) – pick the convention $Z_{base} = V_{LL}^2 / S_{3\phi}$ and stay with it. Forgetting the square on the voltage-base ratio when changing bases. And treating “%Z” as a loss figure – it is a voltage-drop figure at rated current, which is why a 10% transformer can still be 99% efficient at full load.

Summary. Choose S_{base} once and V_{base} per level; convert everything to pu; series, parallel and fault arithmetic go dimensionless; convert back with each level's own base current to prove the answer.

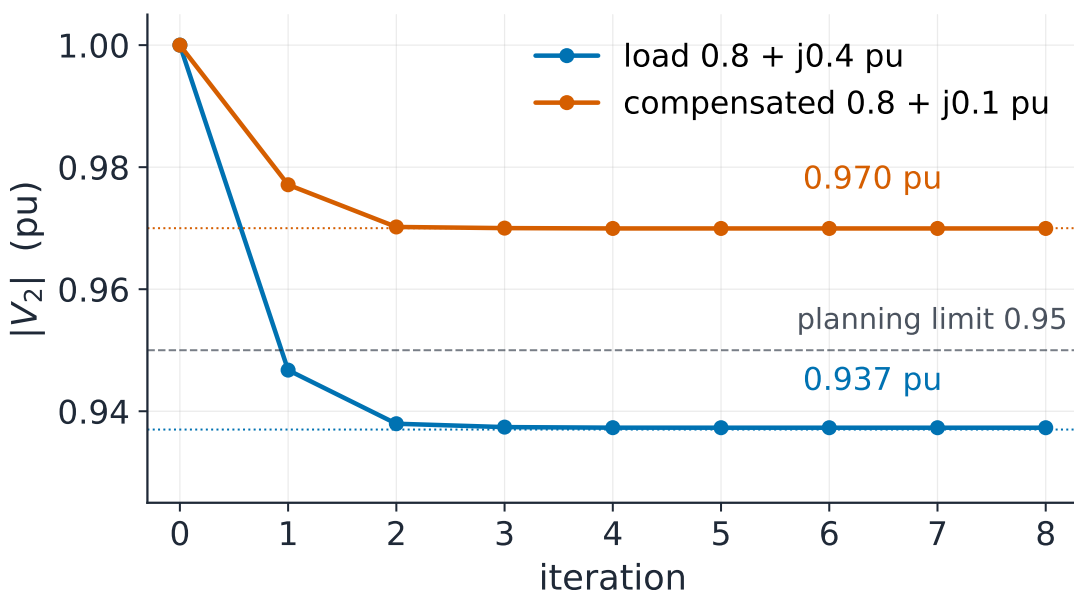
Per-unit and power flow

Why it matters. "Will the voltage at the end of the feeder stay above 0.95?" is the question every grid planner answers thousands of times a day with a **power flow** – and the two-bus version fits on one page.

The idea, with a picture. Bus 1 is the **slack bus** : held at $1.0\angle 0^\circ$ pu, absorbing whatever the losses turn out to be. A line $Z = R + jX$ feeds a load drawing $S = P + jQ$ at bus 2. The load fixes current through its own voltage, and the voltage fixes current through the line – a loop. So guess V_2 , run the loop once, and repeat:

$$V_2^{(k+1)} = V_1 - Z \cdot \text{conj}\left(\frac{S}{V_2^{(k)}}\right),$$

the fixed-point (Gauss–Seidel) iteration. The conjugate is not decoration: with $S = VI^*$, the current is $I = \text{conj}(S/V)$, and skipping it sends the iteration to nonsense.



Guess, correct, repeat: both cases settle within a few iterations – and compensation settles higher

The derivation. From $V_2 = V_1 - ZI$ and $I^* = S/V_2$, each pass computes the current the present V_2 implies and the voltage that current produces. Linearise once around $V \approx 1$ and the famous approximation falls out:

$$\Delta V \approx \frac{PR + QX}{|V_2|},$$

whose second term explains the unit's headline: it is the reactive flow through reactance that eats voltage, which is why capacitors – **reactive compensation**

– are a voltage tool.

Worked example 1 – the base case. $Z = 0.02 + j0.10$ pu, load $0.8 + j0.4$ pu, $V_1 = 1.0\angle 0^\circ$. Starting from $V_2 = 1.0$ the iteration converges to $V_2 = 0.937\angle -4.4^\circ$ pu, with line current $I = 0.954\angle -31^\circ$ pu. Checks: the approximation gives $\Delta V \approx \frac{0.8 \times 0.02 + 0.4 \times 0.10}{0.937} = 0.060$ pu against the exact $1 - 0.937 = 0.063$ ✓ honest at this loading; of the drop, $QX = 0.040$ dwarfs $PR = 0.016$ – reactive dominates. The sending-end balance: $P_s = 0.8 + |I|^2 R = 0.8 + 0.954^2 \times 0.02 = 0.818$ pu ✓ and $Q_s = 0.4 + 0.954^2 \times 0.10 = 0.491$ pu: the line itself eats 0.091 of reactive.

Worked example 2 – compensation. Install capacitors at bus 2 cutting the net load to $0.8 + j0.1$ pu and iterate again: V_2 settles at $0.970\angle -4.6^\circ$ pu. The drop falls from 0.063 to 0.030 pu – the QX term dropped from 0.040 to 0.010 and the voltage followed – while the angle stays near 4.5° , because the angle is carried by P , which did not change. In per-unit the design sentence is one line: 0.3 pu of capacitors at the load lifts V_2 as far as halving the line impedance would (0.970 pu either way), and capacitors cost far less than a second line.

Common mistakes. Dropping the conjugate (the iteration then diverges or converges to a wrong operating point quietly). Letting the slack bus fix more than it should – it sets the angle reference and swallows losses; loads elsewhere must still balance in the converged answer. And trusting $\Delta V \approx (PR + QX)/V$ when the angle across the line grows: the approximation quietly ignores the quadrature component $\frac{PX - QR}{V}$, fine here at 4.4° , misleading past about 15° .

Summary. Slack at one end, load at the other, iterate $V_2 = V_1 - Z \text{conj}(S/V_2)$; verify with the $PR + QX$ approximation and close the power balance. Reactive flow is the voltage problem; compensation is the cheap half of the fix.

Power electronics introduction

Why it matters. The same charger opens with a switch that flips a hundred thousand times a second. Switching converters are why a 30-watt adapter is now smaller than its own plug – and why this unit ends the course: machines meet electronics at the drive.

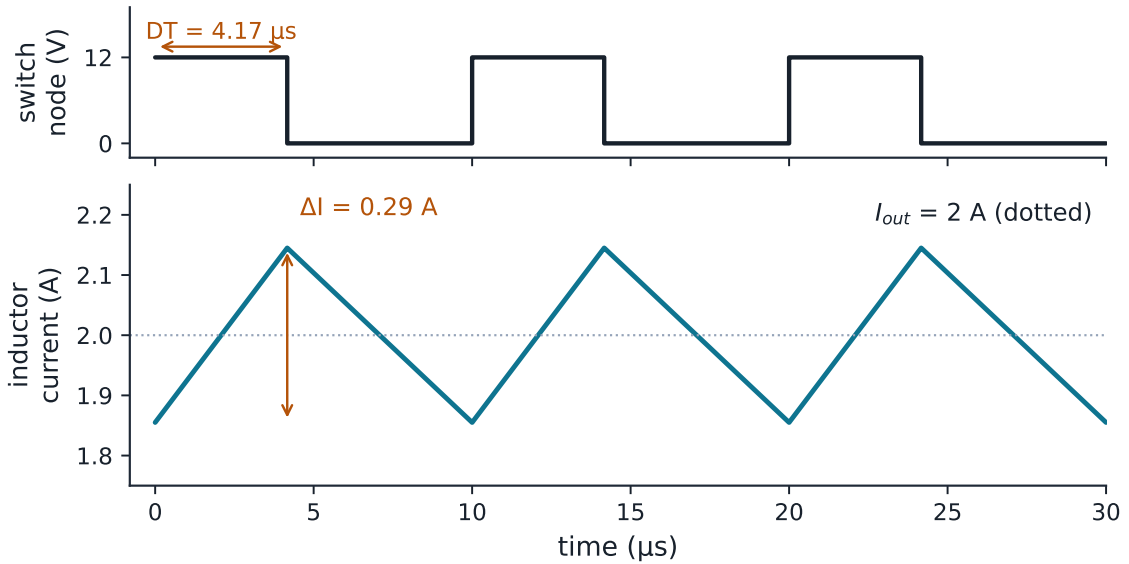
The idea, with a picture. A buck converter is a switch, a diode, an inductor and a capacitor. While the switch is ON the input feeds the inductor; while it is OFF the inductor keeps current flowing through the diode. The inductor and capacitor do the averaging; the switch only chooses the ratio of on-time to period, the **duty cycle**

D. Volt-second balance on the inductor – the average of v_L over a period must be zero in steady state – gives, in one line:

$$(V_{in} - V_{out}) DT = V_{out} (1 - D) T \implies V_{out} = D V_{in},$$

and the **boost** dual gives $V_{out} = \frac{V_{in}}{1 - D}$. **Continuous conduction**

(CCM) means the inductor current never reaches zero – the small-ripple world all these formulas live in.



Inductor current ramps up while the switch is on and down while it is off

The derivation. In **continuous conduction**, the buck ON phase the inductor sees $V_{in} - V_{out}$ for DT seconds; in the OFF phase $-V_{out}$ for $(1 - D)T$. Setting the volt-second sum to zero yields $V_{out} = DV_{in}$ – independent of L , C and load in CCM, which is the converter's charm. The triangular current swing follows from $v = L di/dt$ on either phase:

$$\Delta I_L = \frac{V_{out}(1 - D)}{Lf}, \quad \Delta V_{out} = \frac{\Delta I_L}{8Cf},$$

the ripple pair every datasheet quotes.

Worked example 1 – a 12 V to 5 V buck. Twelve volts in, 5 V at 2 A out, switching at 100 kHz with $L = 100 \mu\text{H}$. Duty $D = \frac{5}{12} = 0.417$. Ripple current $\Delta I_L = \frac{5 \times 0.583}{100\mu \times 100k} = 0.29 \text{ A}$ – 15% of the load, a standard design point. CCM holds because $\Delta I_L/2 = 0.15 \text{ A}$ is far below 2 A. With $C = 220 \mu\text{F}$ the output ripple is $\Delta V_{out} = \frac{0.29}{8 \times 220\mu \times 100k} = 1.7 \text{ mV}$: the converter's output is cleaner than the supply it runs from.

Worked example 2 – a 12 V to 24 V boost. The boost runs the same parts re-arranged: switch across the inductor, diode to the output. For 24 V at 1.5 A from 12 V, $D = 1 - \frac{12}{24} = 0.5$. Power conserved (ideal): $P_{in} = 36$ W, so the input current is 3 A – not 1.5 A; current trades against voltage, power does not. Ripple $\Delta I_L = \frac{V_{in}D}{Lf} = \frac{12 \times 0.5}{100\mu \times 100k} = 0.6$ A, CCM comfortable against 3 A. The output capacitor sees the diode’s chopped current – pulses only during $(1 - D)$ of each period – so boost converters need bigger, lower-ESR output capacitors than bucks of the same power: the ripple RMS, not the average, sizes the part.

Common mistakes. Light load in a “CCM” design: below the boundary current $\Delta I_L/2$ the converter drops into discontinuous conduction and V_{out} rises above DV_{in} . Ignoring the diode drop in low-voltage outputs – 0.4 V in a 5 V rail is 8%, which is why **synchronous rectification** (a MOSFET for the diode) is routine below about 12 V. And equating input and output current because the topology looks symmetric – only power is conserved.

Summary. Volt-second balance: $V_{out} = DV_{in}$ (buck) or $V_{out} = V_{in}/(1 - D)$ (boost) in CCM; ripples $\Delta I_L = \frac{V_{out}(1 - D)}{Lf}$ and $\Delta V_{out} = \frac{\Delta I_L}{8Cf}$. The switch sets the ratio; the passives set the quality; the losses live in the switch and the diode.