7.7 Waveguides and antennas

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS cutoff • waveguide wavelength • Friis equation

Objective

Compute waveguide cutoff and guide wavelength, and apply the Friis link budget – every guide answer checked against the free-space limit and every link budget against the isotropic baseline.

1 Key knowledge and worked examples

■ Cutoff: the pipe's admission price

A rectangular guide (wide dimension a) supports the dominant TE_{10} mode only above $f_c = c/(2a)$. Below cutoff the wave evanesces – it decays, not propagates. The **guide wavelength** stretches beyond free space:

$$\lambda_g = \frac{\lambda}{\sqrt{1-(f_c/f)^2}}$$

■ Worked example: WR-90 at 10 GHz

$a = 22.86mm \rightarrow f_c = 3\times10^8/(2\times0.02286) = 6.56GHz$. At 10 GHz ($\lambda = 30mm$): $\lambda_g = 30/\sqrt{1-0.656^2} = 39.8mm$ – LONGER than free space because the wave zigzags down the guide (the component along the axis is what advances). Checks: as $f \rightarrow \infty$, $\lambda_g \rightarrow \lambda$ (free space) ✓; as $f \rightarrow f_c^+$, $\lambda_g \rightarrow \infty$ (the wave stops advancing) ✓ – the two limits bracket every guide answer.

■ Antennas and the Friis budget

The half-wave dipole: gain 2.15 dBi, input resistance ≈ 73 Ω. **Friis** $P_r/P_t = G_rG_t(\lambda/4\pi R)^2$ – the inverse-square law with antenna gains. **Worked example**: two 10 dBi (G = 10) antennas 1 km apart at 3 GHz ($\lambda = 0.1m$): $10\times10\times(0.1/(4\pi\times1000))^2 = 6.33\times10^{-9}$ – an 82 dB path loss. Check the isotropic baseline: without gains, `(0.1/12566)^2 = 6.33×10^-11` – 20 dB less (exactly the two 10-dBi gains) ✓.

2 Practice

2.1 A guide with $a = 47.55mm$ (WR-187): compute the cutoff frequency of the dominant mode. [2]

2.2 WR-90 ($f_c = 6.56GHz$) at 12 GHz ($\lambda = 25mm$): compute λ_g. [3]

2.3 Two isotropic antennas 100 m apart at 1.5 GHz: compute the path loss in dB. [3]

3 Exam-style questions

3.1 A rectangular guide $a = 15cm$ is filled with a dielectric ($\varepsilon_r = 2.1$, $\mu_r = 1$).

Compute the cutoff frequency of the TE_{10} mode (remember $v = c/\sqrt{\varepsilon_r}$ in the dielectric), the guide wavelength at 1.5 GHz, and the wave's phase velocity $v_p = f \cdot \lambda_g$ – explain why v_p exceeds the dielectric's light speed and state what the group (energy) velocity does instead. Verify both limits ($f \to f_c^+$ and $f \gg f_c$). [7]

3.2 A satellite link: 12 GHz downlink, satellite EIRP 50 dBW (that is, $P_tG_t = 10^5 W$ effective), receive dish gain 45 dBi, range 38,000 km.

Compute the free-space path loss at 12 GHz ($\lambda = 25$ mm) in dB, the received power in dBW, and in watts. Then compute the noise floor for a 20 MHz bandwidth receiver at 100 K system temperature (`N = kTB`, k $= 1.38\times10^{-23}$), and the link margin in dB. State whether the link closes with a 3 dB requirement for rain fade. [7]