

7.4 Maxwell’s equations

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS displacement current

Objective

Assemble the four Maxwell equations, explain displacement current through the charging capacitor, and use Faraday’s law to compute induced EMF – every answer checked against continuity and the static limits.

1 Key knowledge and worked examples

■ The four equations

$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0 \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \partial \mathbf{E}/\partial t$$

Gauss for E (charge is the source of divergence), Gauss for B (no monopoles), **Faraday** (changing B curls E), Ampère–Maxwell (current PLUS **displacement current** curls B).

■ Worked example: the capacitor closes the loop

A capacitor (plates A) charging at $dQ/dt = 1A$: a conduction current of 1 A flows in the wires, zero between the plates. Ampère’s loop around the wire must read 1 A whether the surface it spans cuts the wire or passes BETWEEN the plates. The displacement current supplies the difference: $E = Q/(\epsilon_0 A)$, so $I_d = \epsilon_0 A \cdot \partial E/\partial t = dQ/dt = 1A$ ✓ – exactly the missing ampere, with no free charge between the plates. Continuity is preserved: $\nabla \cdot (J + \epsilon_0 \partial E/\partial t) = 0$ even while $\nabla \cdot J = -\partial \rho/\partial t$ at the plate.

■ Faraday: flux change induces EMF

$EMF = -d\Phi/dt$ (the minus sign is Lenz: the induced current opposes the change). Worked example: a 100-turn coil of area 10 cm^2 in a field rising linearly $0 \rightarrow 50\text{ mT}$ in 20 ms: $d\Phi/dt$ per turn = $(0.05/0.02) \times 10^{-3} = 2.5\text{ mWb/s}$, so $EMF = 100 \times 2.5 = 250\text{ mV}$. No change means no EMF; faster change, proportionally more.

2 Practice

2.1 Name each Maxwell equation in words and state its static limit (what it reduces to when all time derivatives are zero). [4]

2.2 A parallel-plate capacitor ($A = 0.01\text{ m}^2$) charges at 2 A: compute the displacement current between the plates. [2]

2.3 A single-loop coil of area 25 cm^2 sits in B falling from 80 mT to zero in 40 ms: compute the EMF. [3]

3 Exam-style questions

3.1 A circular parallel-plate capacitor (radius 5 cm, separation 1 mm) charges at a constant $I = 0.5A$.

Compute E(t) between the plates (assume uniform), the displacement current density, and B(r) between the plates at radius r (Ampère–Maxwell on a circular path): show $B(r) = \mu_0 I dr/(2\pi R^2)$ inside the plate radius R. Evaluate B at $r = R$ for this capacitor, and verify the magnitude against the wire’s field just outside (same current) – the two must match at the plate edge. [7]

3.2 A conducting rod of length 20 cm slides at 4 m/s on rails perpendicular to a uniform 0.8 T field (the rail generator).

Compute the motional EMF. The rails close through a $2\ \Omega$ resistor: find the current, the retarding force on the rod, and the mechanical power needed to keep it moving – then verify that power equals the resistor’s dissipation (energy conservation, the check that closes every such problem). State the direction of the induced current (Lenz). [7]
