Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

a coaxial cylinder of radius r and length L; no flux through its ends; $E \cdot 2\pi r L = \dfrac{\lambda L}{\varepsilon_0}$; $E = \dfrac{2.0 \times 10^{-9}}{2\pi \times 8.854 \times 10^{-12} \times 0.050} = \mathbf{719\ V/m}$, away from the line

2.2

one sheet: $E = \dfrac{\sigma}{2\varepsilon_0} = 2.82 \times 10^5$ V/m; at B the two fields add: $\mathbf{5.65 \times 10^5\ V/m}$, from $+$ to $-$; at A and C they cancel: **0**

2.3

$C = \dfrac{\varepsilon_r \varepsilon_0 A}{d} = \dfrac{4.7 \times 8.854 \times 10^{-12} \times 1.0 \times 10^{-4}}{1.0 \times 10^{-4}} = \mathbf{41.6\ pF}$; $W = \frac{1}{2} \times 41.6 \times 10^{-12} \times 5.0^2 = \mathbf{520\ pJ}$

2.4

tangential part continuous: $E_{2t} = 1000 \sin 60^\circ = 866$ V/m; normal D continuous: $E_{2n} = \dfrac{1000 \cos 60^\circ}{3.0} = 167$ V/m; $E_2 = \sqrt{866^2 + 167^2} = \mathbf{882\ V/m}$ at $\tan^{-1} \dfrac{866}{167} = \mathbf{79}^\circ$ to the normal

3.1

Gauss on a coaxial cylinder: $E = \dfrac{\lambda}{2\pi\varepsilon r}$ for $a < r < b$

$V = \dfrac{\lambda}{2\pi\varepsilon} \ln \dfrac{b}{a}$; $C' = \dfrac{2\pi \times 2.3 \times 8.854 \times 10^{-12}}{\ln 4} = \mathbf{92.3\ pF/m}$

$E = \dfrac{V}{r \ln(b/a)}$ is largest at $r = a$: $\dfrac{100}{0.0010 \times \ln 4} = \mathbf{72\ kV/m}$; at the surface of the inner conductor, where breakdown starts first

3.2

image $-q$ at 5.0 cm below the plane; the field above is that of q and $-q$

at P both fields point down: $E = 2 \times \dfrac{q}{4\pi\varepsilon_0 h^2} = 2 \times \dfrac{8.99 \times 10^9 \times 10 \times 10^{-9}}{0.050^2} = \mathbf{7.2 \times 10^4\ V/m}$, into the plane

$\sigma = -\varepsilon_0 E = \mathbf{-0.64\ \mu C/m^2}$; $F = \dfrac{q^2}{4\pi\varepsilon_0 (2h)^2} = \dfrac{8.99 \times 10^9 \times (10^{-8})^2}{0.10^2} = \mathbf{9.0 \times 10^{-5}\ N}$, towards the plane

3.3

(a) $V = \dfrac{Q}{4\pi\varepsilon_0 a}$ and $E = \dfrac{Q}{4\pi\varepsilon_0 a^2}$, so $E = \dfrac{V}{a}$; $E = \dfrac{30 \times 10^3}{0.010} = \mathbf{3.0\ MV/m}$: just at breakdown

(b) $C = 4\pi\varepsilon_0 a = \mathbf{1.11\ pF}$; $Q = CV = \mathbf{33\ nC}$

(c) need $E \le 1.5$ MV/m, so $a \ge \dfrac{30 \times 10^3}{1.5 \times 10^6} = \mathbf{2.0\ cm}$

(d) the field near a conductor goes roughly as V over the local radius of curvature, so a sharp edge has a much higher field and starts **corona** there; round off every edge, or hide the screw inside a smooth larger shape such as a corona ring

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