

6.1 Signal representations and system properties

Name: _____ Class: _____ Date: _____

Total: 22 marks

KEY WORDS signal • system • linearity • time invariance
 • causality • BIBO stability BIBO

Objective

Classify signals and systems by the four properties, apply the shift-then-system versus system-then-shift test, and verify LTI behaviour from the definition.

1 Key knowledge and worked examples

■ Four properties, tested from the definition

- **Linearity** : $a \cdot x_1 + b \cdot x_2 \rightarrow a \cdot y_1 + b \cdot y_2$ – test with superposition.
- **Time invariance** : shift input by $t_0 \Rightarrow$ output shifts by exactly t_0 – compare "shift then system" with "system then shift".
- **Causality** : no output before input – check $y(t)$ for $t < 0$ on an input starting at 0.
- **BIBO stability** BIBO : bounded in, bounded out.

■ Worked example: the time-scaling trap

Is $y(t) = x(2t)$ time-invariant? Input $u(t)$: $y = u(2t) = u(t)$ (same step). Shift the input: $x(t-1) = u(t-1)$ gives $y = u(2t-1) = u(t-\frac{1}{2})$. But shifting the original OUTPUT gives $y(t-1) = u(2(t-1)) = u(2t-2) = u(t-1)$. Half shift versus full shift – different: **time-varying** ✓ (time scaling rescales shifts). Systems that are memoryless and of the form $y = f(x(t))$ with fixed f ARE time-invariant; it is the argument scaling that breaks the property.

■ Stability from the impulse response

For LTI systems, BIBO $\Leftrightarrow \int |h(t)|dt < \infty$ (the impulse response is absolutely integrable). Worked checks: $h = e^{-t}u(t)$ – integrable, stable ✓; $h = u(t)$ (an integrator) – the integral diverges, unstable (a ramp input diverges) ✓; $h = \sin(t)/t$ – NOT absolutely integrable, yet bounded outputs: the subtlety that motivates the strict

definition. Causality from $\mathbf{h}$: $\mathbf{h}(\mathbf{t}) = 0$ for $\mathbf{t} < 0$.

2 Practice

2.1 Classify each as linear/nonlinear: $y = 3x + 1$; $y = t \cdot x(t)$; $y = x^2$. [3]

2.2 Is $y(t) = x(t) + 5$ time-invariant? Causal? [2]

2.3 Which impulse responses are BIBO-stable: $e^{-2t}u(t)$, $\mathbf{u}(\mathbf{t})$, $e^{-t}\cos(t)u(t)$? [3]

3 Exam-style questions

3.1 Test each system on all four properties (linearity, time invariance, causality, BIBO stability), with one-line evidence each:

(a) $y(t) = x(t - 2)$; (b) $y(t) = t \cdot x(t)$; (c) $y(t) = \int_{-\infty}^t x(\tau)d\tau$; (d) $y[n] = x[n]^2$. [8]

3.2 A system is claimed LTI. Two measurement pairs: $x_1 = \delta(t) \rightarrow y_1 = e^{-t}u(t)$; $x_2 = \delta(t-1) \rightarrow y_2 = e^{-(t-1)}u(t-1)$.

Verify the second response is the LTI-predicted one. Then predict the response to $x_3 = 2\delta(t) + 3\delta(t-1)$ and check the claimed system's BIBO stability from its impulse response. State whether $y(t) = x(t) \cdot h(t)$ would be a valid output rule for this system (and why not). [6]
