

4.7 Two-port networks

Name: _____ Class: _____ Date: _____

Total: 22 marks

KEY WORDS two-port network • cascade

Objective

Compute z , y and ABCD parameter sets for two-ports, convert between them, apply terminated-load formulas, and cascade ABCD matrices – every conversion verified by the reciprocal-network determinant test.

1 Key knowledge and worked examples

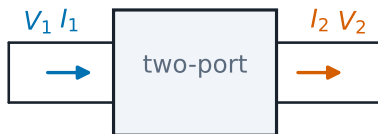
■ Three parameter sets

A two-port relates (V_1, I_1) to (V_2, I_2) :

- **z-parameters** z : $V_1 = z_{11}I_1 + z_{12}I_2$, $V_2 = z_{21}I_1 + z_{22}I_2$ – impedances, found by open-circuit tests ($I_2 = 0$: $z_{11} = V_1/I_1$, etc.).
- **y-parameters**: the dual (short-circuit admittances).
- **ABCD** (transmission): $V_1 = AV_2 - BI_2$, $I_1 = CV_2 - DI_2$ – cascades multiply.

For a reciprocal network: $z_{21} = z_{12}$ and $AD - BC = 1$ – the standing check for every conversion.

port conventions



ABCD: current OUT of port 2

elementary ABCD blocks

series Z $(1, Z; 0, 1)$
impedance in the line

shunt Y $(1, 0; Y, 1)$
admittance across

cascade = matrix multiply

Port conventions and the elementary ABCD blocks every chain composes.

■ Worked example: the L-section, cascaded

Series $Z = j10$ has $ABCD = (1, Z; 0, 1)$; shunt $Y = -j0.1$ has $(1, 0; Y, 1)$. Cascading shunt- Y then series- Z :

$(1, 0; Y, 1)(1, Z; 0, 1) = (1, Z; Y, 1 + YZ)$ – with $Z = j10, Y = -j0.1$: $1 + YZ = 1 + 1 = 2$, so the product is $(1, j10; -j0.1, 2)$. Check: $AD - BC = 1 \times 2 - j10 \times (-j0.1) = 2 - 1 = 1$ ✓ – reciprocal, as pure L/C networks must be.

■ Terminated two-ports

With a load Z_L on port 2: $Z_{in} = (AZ_L + B)/(CZ_L + D)$ and the voltage gain $V_2/V_s = Z_L/(AZ_L + B)$ (source impedance included in the ABCD if present). **Worked example:** the L-section above loaded by $Z_L = 2\Omega$: $Z_{in} = (1 \times 2 + j10)/(-j0.1 \times 2 + 2) = (2 + j10)/(2 - j0.2)$. Multiply by the conjugate: $(2 + j10)(2 + j0.2)/(4 + 0.04) = (4 - 2 + j(0.4 + 20))/4.04 = (2 + j20.4)/4.04 = 0.495 + j5.05 \Omega$. Check: at this frequency the shunt C and load R mostly parallel the series L – the input is dominated by $j10$ minus the shunt's small resistive dip ✓ plausible; magnitude $5.07\Omega < |j10|$ ✓ (the shunt pulls impedance down).

2 Practice

2.1 For the series impedance two-port $Z = 5\Omega$: write its ABCD matrix and verify $AD - BC = 1$. [2]

2.2 A two-port has $z_{11} = 10, z_{12} = 4, z_{21} = 4, z_{22} = 6\Omega$ (all resistive). Is it reciprocal? Verify. [2]

2.3 Convert $z = (10, 4; 4, 6)$ to ABCD ($A = z_{11}/z_{21}$, $B = \Delta z/z_{21}$, $C = 1/z_{21}$, $D = z_{22}/z_{21}$ with $\Delta z = z_{11}z_{22} - z_{12}z_{21}$). Verify $AD - BC = 1$. [4]

3 Exam-style questions

3.1 A resistive T-network has series arms $8\ \Omega$ and $12\ \Omega$ with a $4\ \Omega$ shunt in the middle.

Compute the z-parameters by open-circuit tests (drive port 1 with $I_2 = 0$: z_{11} and z_{21} ; then the reverse). Check reciprocity, convert to ABCD, and verify $AD - BC = 1$. [7]

3.2 Cascade two sections, each $ABCD = (1, 10; 0.02, 1.2)$.

Compute the overall ABCD, then the input impedance with a $50\ \Omega$ load ($Z_{in} = (AZ_L + B)/(CZ_L + D)$), and verify the overall AD - BC equals 1 given each section satisfies it (prove the product property: $(A_1D_1 - B_1C_1)(A_2D_2 - B_2C_2) = 1$ implies the product matrix also has unit determinant). [7]
