

4.6 Laplace circuit analysis

Name: _____ Class: _____ Date: _____

Total: 22 marks

KEY WORDS Laplace transform • pole

Objective

Transform circuits with initial conditions into the s-domain, solve by all the DC methods, and read the response – poles, transients and steady state – from the algebra, verifying against time-domain endpoints.

1 Key knowledge and worked examples

■ The s-domain element models

The **Laplace transform** turns each element into an impedance plus initial-condition sources:

- Resistor: $Z = R$ (no initial energy).
- Inductor with $i(0^-)$: $Z = sL$ in series with a voltage source $L \cdot i(0^-)$ (or parallel current source $i(0^-)/s$).
- Capacitor with $v(0^-)$: $Z = 1/(sC)$ in parallel with a current source $C \cdot v(0^-)$ (or series voltage source $v(0^-)/s$).

Then dividers, node and mesh all apply in s. Poles of the response = transient exponents; the input's poles = steady state.

The s-domain models: impedance plus initial-condition source

Resistor $Z = R$ no initial energy — nothing extra

Inductor $Z = sL$ plus series source $Li(0^-)$

Capacitor $Z = 1/(sC)$ plus parallel source $Cv(0^-)$

Then every DC method (divider, mesh, Thévenin) applies in s — with the sources carrying the past

The s-domain models: impedance plus initial-condition source.

■ Worked example: the RC step, with initial charge

$R = 10k\Omega$, $C = 100\mu F$ ($\tau = 1$ s), step 9 V, $v_C(0) = 1V$.

- s-domain KVL: $9/s = I \cdot R + v_C(0)/s + I/(sC)$.
- $I = (9 - 1)/(s(R + 1/(sC))) = 8/(10^4 s + 10^4) = 8/(10^4(s + 1))$.
- $V_C = v_C(0)/s + I/(sC) = 1/s + 8/(s(s + 1)) = 9/s - 8/(s + 1)$.
- $v_C(t) = 9 - 8e^{-t}$ V – the wave-1 answer, now algebra.

Checks: $v_C(0) = 1$ ✓; $v_C(\infty) = 9$ ✓; the single pole at -1 gives the 1-second time constant ✓. The identity $1/s + 8/(s(s + 1)) = 9/s - 8/(s + 1)$ verifies at any s (e.g. $s = 1$: $1 + 4 = 5 = 9 - 4$ ✓).

■ What the poles say

A response $V(s)$ with poles at $s = 0$ (constant), $s = -a$ (decay e^{-at}), and $\pm j\omega$ (oscillation) reads its whole time shape before inversion. Initial conditions scale the residues; they never move the poles – the poles belong to the circuit, the residues to its history.

2 Practice

2.1 Transform each: a $4\ \Omega$ resistor; a 0.5 H inductor (no initial current); that inductor with $i(0) = 2$ A (impedance + series source value). [3]

2.2 A 2 F capacitor with $v(0) = 3$ V: give the impedance and the parallel current source value. [2]

2.3 $F(s) = 10/s - 10/(s+2)$: give $f(t)$ and its two endpoint values. [3]

3 Exam-style questions

3.1 A series RL circuit ($R = 4\Omega$, $L = 0.2H$) with $i(0) = 1.5$ A is driven at $t = 0$ by a 24 V step.

Transform to the s-domain, solve for $I(s)$, and invert to $i(t)$. Verify $i(0)$ and $i(\infty)$, and give the time constant from the pole. Then recompute the steady state directly from the circuit (inductor as short) as an independent check. [7]

3.2 A capacitor $C = 0.5F$ charged to $v(0) = 10$ V discharges through $R = 2\Omega$ (no source).

Using the s-domain model (impedance + initial-condition source), solve for the capacitor voltage $V_C(s)$ and invert. Verify $v(0)$ and $v(\infty)$, give the pole, and state the energy dissipated in the resistor over all time (check against $\frac{1}{2}CV^2$). [7]
