4.5 Filters and Bode plots

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS filter • Bode plot • corner frequency • decibel

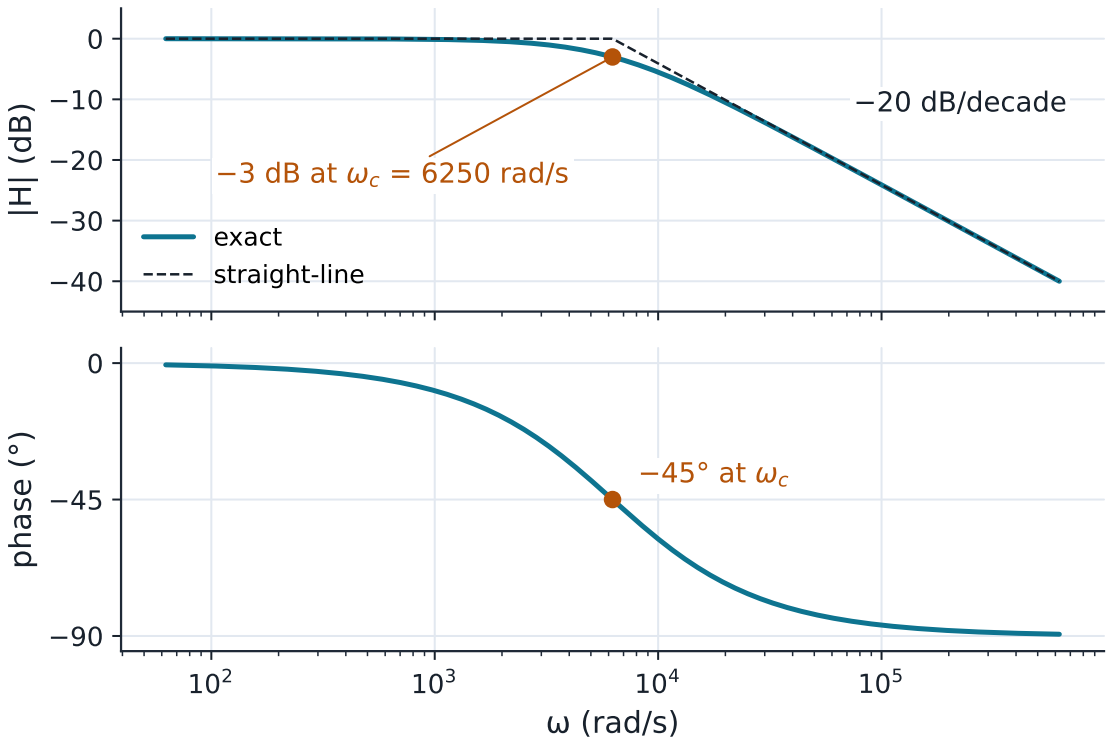
Objective

Design and analyse first/second-order passive filters, sketch straight-line Bode asymptotes with correctly labelled corners, and verify designs against stopband specifications in dB.

1 Key knowledge and worked examples

■ Filter classes and the transfer function

A **filter** transfer function $H(j\omega) = V_{out}/V_{in}$ classifies by its passband: low-pass, high-pass, band-pass (series RLC take-off across R), band-stop (across LC). First-order passive sections roll off at 20 dB/decade; two cascaded (first-order) sections give 40 dB/decade.



The RC low-pass: −3 dB and −45° at the corner, −20 dB per decade after.

■ The Bode plot: straight lines that carry the design

The **Bode plot** plots $20log_{10}|H|$ (dB) against $\log \omega$. A first-order low-pass RC has magnitude $1/\sqrt{(1+(\omega/\omega_c)^2)}$ and the **corner frequency** $\omega_c = 1/(RC)$:

- $\omega \ll \omega_c$: 0 dB (flat); $\omega = \omega_c$: -3 dB, phase $-45°$; $\omega \gg \omega_c$: -20 dB/decade, phase $\to -90°$.

Worked example. $R = 1.6k\Omega$, $C = 0.1\mu F$: $\omega_c = 6250rad/s$ (995 Hz). At $10\omega_c$ the exact gain is $1/\sqrt{101} = -20.04dB$ – the straight line reads -20 dB. Each pole -20 dB/decade; each zero $+20$; dB arithmetic is $20log_{10}10 = 20$ per decade.

■ Designing against a stopband spec

Worked example. Require at least -40 dB at 10 kHz with an RC low-pass: -40 dB needs $(\omega/\omega_c)^2 \geq 10^4 \to \omega_c \leq \omega/100$. So a single section needs $\omega_c \leq 2\pi \times 10^4/100 = 628rad/s$. Check: at $\omega_c = 628$, gain at 10 kHz is $1/\sqrt{1+100^2} = 1/100.005 \to -40.0$ dB ✓. Two cascaded sections would allow a higher corner (-40 dB/decade), trading simplicity for passband width.

2 Practice

2.1 An RC low-pass has R = 2 kΩ, C = 0.05 µF. Compute ω_c in rad/s and Hz, and the phase at ω_c. [3]

2.2 For that filter, give the gain in dB at ω_c, at $10\omega_c$ (exact to 1 dp), and at $100\omega_c$ (asymptotic). [3]

2.3 A first-order high-pass RC (output across R) has the same R and C. State its corner and its asymptotic slope at low frequency. [2]

3 Exam-style questions

3.1 Design a first-order RC low-pass that attenuates a 4 kHz tone by at least 30 dB while passing 400 Hz with no more than 1 dB loss.

Derive the corner constraints from BOTH specifications, choose a corner (with R = 3.3 kΩ, compute C), and verify the design meets both specs exactly. State the conflict if the two constraints leave no feasible corner. [7]

3.2 A band-pass RLC (series R, output across R) has $R = 50\Omega$, $L = 25mH$, $C = 0.1\mu F$.

Compute ω_0, $\mathtt{Q}$, the bandwidth, and the two half-power frequencies. Sketch (describe) the Bode magnitude asymptotes: flat 0 dB between the corners, falling −20 dB/decade below the lower and above the upper corner. Verify the peak gain is 0 dB (all source voltage across R at resonance). [7]