4.4 Resonance

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS resonance • bandwidth • Q factor

Objective

Compute resonant frequency, bandwidth and Q for series and parallel RLC circuits; explain the impedance minimum/maximum; and verify resonant voltage/current magnifications numerically.

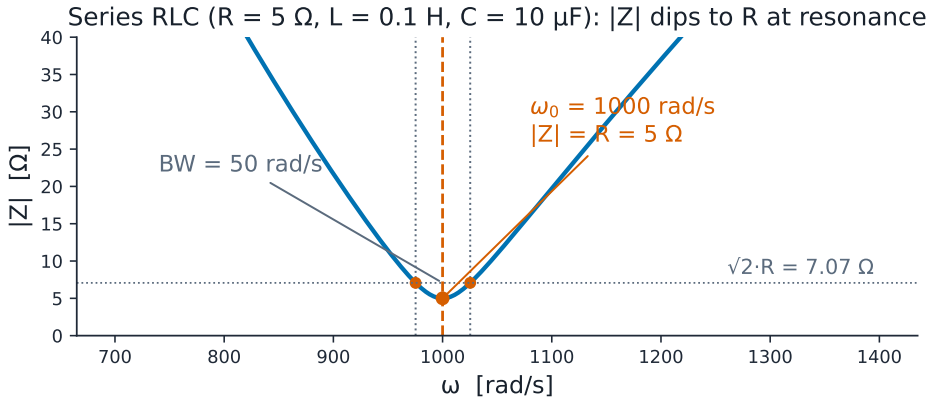
1 Key knowledge and worked examples

■ Resonance: the imaginary parts cancel

A series RLC is at **resonance** when $\omega_0 L = 1/(\omega_0 C)$:

$$\omega_0 = \frac{1}{\sqrt{LC}}, \qquad BW = \frac{R}{L} = \frac{\omega_0}{Q}, \qquad Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$$

At ω_0: `Z = R` (purely resistive, minimum magnitude) → maximum current; the **bandwidth** BW is the span between the half-power frequencies. In the PARALLEL RLC the dual holds: impedance is MAXIMUM at resonance.



|Z| dips to R at ω_0; the half-power band is marked.

■ Worked example: voltage magnification

$R = 5\Omega$, $L = 0.1H$, $C = 10\mu F$: $\omega_0 = 1/\sqrt{10^{-6} \times 0.1} = 1000 rad/s$, $Q = 1000 \times 0.1/5 = 20$.

At resonance with a 10 V source: $I = 10/5 = 2A$; $V_L = \omega_0 L I = 200V$ and $V_C = I/(\omega_0 C) = 200V$ – twenty times the source, 180° apart, cancelling each other while each alone is huge. That is what Q means, measured in volts.

■ Selectivity and the frequency sweep

A frequency sweep past resonance shows the current peaking at ω_0 and falling on both sides at $\pm$BW/2 to $1/\sqrt{2}$ of peak. High Q = narrow peak = a selective tuner; the radio tuning dial IS this arithmetic – a station spacing of `~200 kHz` at `~100 MHz` needs $Q \approx 500$, and $Q = \omega_0 L/R$ tells you how small R must be.

2 Practice

2.1 $L = 0.2H$, $C = 20\mu F$: compute ω_0 in rad/s and Hz. [2]

2.2 A series RLC has $R = 10\Omega$, $\omega_0 = 500 rad/s$, $Q = 25$. Compute L, C and the bandwidth. [3]

2.3 At resonance the series circuit of 2.2 is driven by 5 V. Compute the current, V_L and V_C. [3]

3 Exam-style questions

3.1 A series RLC has $R = 2\Omega$, $L = 0.05H$, $C = 0.8\mu F$.

Compute ω_0, Q, the bandwidth, and the half-power frequencies. With a 15 V source at resonance, compute the current and the capacitor voltage, and verify the magnification factor equals Q. Then state (with the reason) what happens to the bandwidth if R doubles with L, C unchanged. [7]

3.2 A parallel RLC ($R = 40k\Omega$ in parallel with $L = 0.4mH$ and $C = 100nF$) is driven by a 2 mA source.

Compute ω_0 and the impedance at resonance; compute the Q for the parallel circuit ($Q = R/(\omega_0 L)$) and the voltage across the tank at resonance. Verify that the tank current magnitudes in L and C each equal Q $\times$ source current, and explain why the line current is only 2 mA while the branch currents are much larger. [7]