UNIVERSITY ELECTRICAL ENGINEERING • EXERCISE SHEET

4.3 Second-order RLC circuits

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS natural frequency • damping • underdamped • Q factor • overshoot • settling time

Objective

Classify second-order responses by the damping ratio, compute step-response specifications, and verify every solution against its initial and final values.

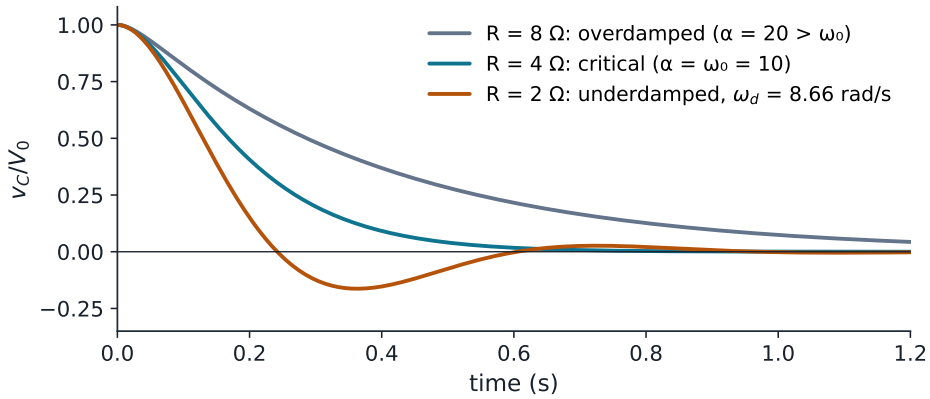
1 Key knowledge and worked examples

■ **Two numbers decide everything**

The series RLC natural response is governed by the **natural frequency** $\omega_0 = 1/\sqrt{LC}$ and the **damping** $\alpha = R/(2L)$:

- **Overdamped** ($\alpha > \omega_0$): $Ae^{s_1 t} + Be^{s_2 t}$, two real decays.
- **Critically damped** ($\alpha = \omega_0$): $(A+Bt)e^{-\alpha t}$ – fastest without oscillation.
- **Underdamped** ($\alpha < \omega_0$): $e^{-\alpha t}(Acos\omega_d t + Bsin\omega_d t)$, ringing at $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$.

The **Q factor** $Q = \omega_0/(2\alpha)$ measures the ring persistence.



One circuit, three resistors: overdamped, critical, underdamped.

■ **Worked example: one circuit, two regimes**

$L = 0.2H$, $C = 0.05F$: $\omega_0 = 1/\sqrt{0.01} = 10rad/s$.

- $R = 4\Omega$: $\alpha = 4/0.4 = 10 = \omega_0$ – critically damped.
- $R = 2\Omega$: $\alpha = 5 < 10$ – underdamped, $\omega_d = \sqrt{100 - 25} = 8.66rad/s$, $Q = 10/10 = 1$.

The regime test is one comparison – but the check that earns marks is the endpoint check: at $t = 0$ the response must equal the initial condition, and `di/dt(0)` must match what the circuit allows (for an inductor current, zero slope at switch-on when driven from a voltage step through R).

■ **Step-response specifications**

For the underdamped step response: **overshoot** $M_p = e^{-\pi\alpha/\omega_d}$ (as a fraction); **settling time** (2%) $t_s \approx 4/\alpha$; rise time grows with damping. **Worked example**: with $\alpha = 5, \omega_d = 8.66$: $M_p = e^{-5\pi/8.66} = e^{-1.814} = 0.163$ (16.3%), $t_s \approx 0.8s$. Check M_p bounds: $0 <$ M_p < 1 for any $\alpha > 0$ ✓; and $cos\omega_d t_p =$ ... the peak time $t_p = \pi/\omega_d = 0.363s$, inside the settling window – consistent ordering.

2 Practice

2.1 $R = 60\Omega$, $L = 1H$, $C = 1/400F$: compute ω_0, α, name the regime. [3]

2.2 Compute ω_d for the circuit of 2.1 if it is underdamped (or state the regime result instead). [2]

2.3 An underdamped system has $\alpha = 2, \omega_0 = 10$. Compute Q, M_p and the 2% settling time. [3]

3 Exam-style questions

3.1 A series RLC with $R = 5\Omega$, $L = 0.5H$, $C = 0.02F$ starts relaxed and is driven at $t = 0$ by a 20 V step.

Compute ω_0, α, the regime, and the complete capacitor-voltage response $v_C(t)$. Verify against $v_C(0)$ and $dv_C/dt(0)$ (zero, for a relaxed capacitor) and the final value. [8]

3.2 A control designer needs the second-order step response to settle (2%) within 0.5 s with overshoot below 10%.

Derive the required α from the settling spec and the minimum α/ω_d from the overshoot spec ($M_p = e^{-\pi\alpha/\omega_d}$), then choose α and the constraint on ω_d (or ω_0) for a system with $\omega_0 = 20 rad/s$ available. Verify your design computes M_p and t_s on target. [6]