

4.1 Sinusoidal steady state and phasors

Name: _____ Class: _____ Date: _____

Total: 22 marks

KEY WORDS phasor • impedance

Objective

Convert sinusoids to phasors, compute impedances, and solve AC circuits with all the DC methods – every answer verified in both polar and rectangular form.

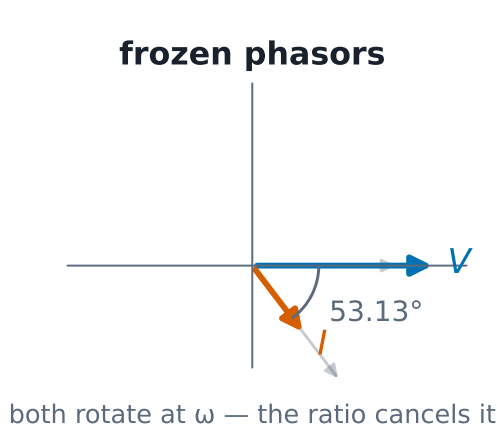
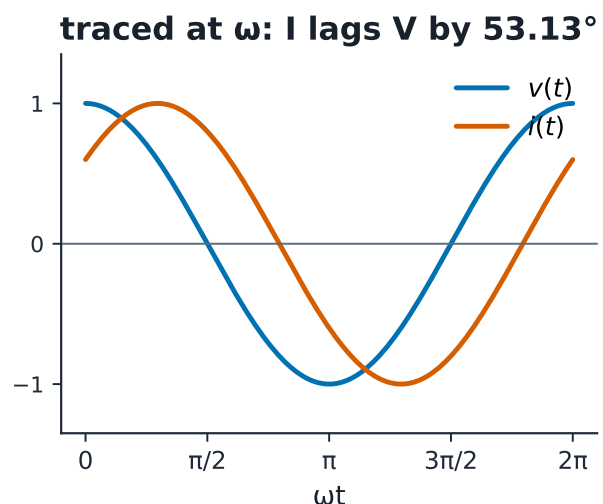
1 Key knowledge and worked examples

■ The phasor transformation

$v(t) = V_m \cos(\omega t + \varphi)$ becomes the **phasor** $V = V_m \angle \varphi$. At one frequency, every element is an **impedance** :

- $Z_R = R$ (0°), $Z_L = j\omega L$ ($+90^\circ$), $Z_C = 1/(j\omega C)$ (-90°).

Impedances combine exactly like resistors: series adds, parallel is product-over-sum. Ohm's law, dividers, node and mesh analysis all carry over.



The rotating picture behind the frozen phasors: ratios cancel the rotation.

■ Worked example: the RL load, both forms

$R = 30\Omega$ in series with $\omega L = 40\Omega$, driven by $V = 100\angle 0^\circ V$.

- Polar: $Z = \sqrt{30^2 + 40^2} \angle \text{atan}(40/30) = 50\angle 53.13^\circ$; $I = 100/50 \angle (0 - 53.13^\circ) = 2\angle -53.13^\circ A$.
- Rectangular check: $I = 100(30 - j40)/(30^2 + 40^2) = (3000 - j4000)/2500 = 1.2 - j1.6 A$; $|I| = 2 \checkmark$ and $\text{atan2}(-1.6, 1.2) = -53.13^\circ \checkmark$.

Current lags voltage by 53.13° – the inductor’s mark, visible in the phasor before any arithmetic on the time domain.

■ Time-domain round trip

$i(t) = 2\sqrt{2}\cos(\omega t - 53.13^\circ) A$ if the phasor used RMS amplitude convention $V = 100V$ RMS – state which convention you use and stay in it. The final check is dimensional: the phasor method predicts the measured amplitude AND phase of every branch voltage, which is what an oscilloscope shows.

2 Practice

2.1 Convert to polar: $Z = 8 + j15$, $Z = 12 - j5$. [2]

2.2 A 40Ω resistor in series with a capacitor whose reactance is 30Ω at the operating frequency: find Z , and I for $V = 80\angle 0^\circ V$. [3]

2.3 Two impedances $Z_1 = 6 + j8$ and $Z_2 = 12 - j4$ are in parallel. Find the equivalent impedance (verify by computing $Z_1 Z_2 / (Z_1 + Z_2)$ both ways). [4]

3 Exam-style questions

3.1 A circuit has $R = 10\Omega$ in series with L ($\omega L = 20\ \Omega$) and C (reactance $10\ \Omega$ at the same ω), driven by $V_s = 60\angle 30^\circ V$.

Compute the total impedance, the current phasor, and the voltage across the capacitor (as a phasor, polar form). Verify the loop: the sum of the three element voltage phasors returns V_s . [7]

3.2 A phasor-domain nodal analysis: node A is fed by $V_s = 50\angle 0^\circ V$ through $Z_1 = j10\Omega$, and connected to ground through $Z_2 = -j10\Omega$ (a capacitor at this frequency) and, in parallel with it, $Z_3 = 5\Omega$.

Write the single node equation, solve for V_A , and compute the current in each of the three branches. Verify KCL at node A. [6]
