

2.3 Linear algebra essentials

Name: _____ Class: _____ Date: _____

Total: 23 marks

KEY WORDS linear system

• matrix inverse

• determinant

• singular

Objective

Solve 2×2 and 3×3 linear systems by elimination, invert a 2×2 matrix, detect singularity from the determinant, and verify every solution by substitution into the unused equations.

1 Key knowledge and worked examples

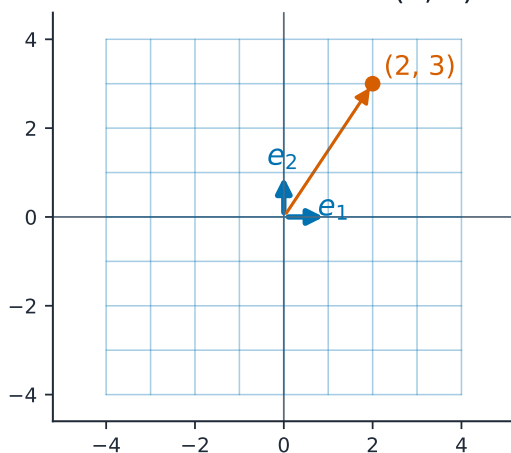
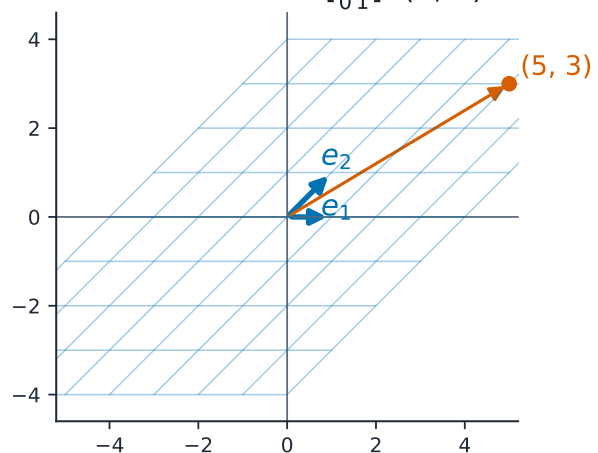
■ Elimination, and what the determinant says

A **linear system** $Ax = b$ is solved by elimination: use one equation to remove a variable from another. The **determinant** of a 2×2 matrix,

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc,$$

is non-zero exactly when the rows are independent – a **singular** system ($\det = 0$) has no unique solution: in circuits, a floating node or a loop of dependent sources.

Coordinates change with the basis; area does not
the standard basis: (2, 3) after the shear $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$: (5, 3)



■ Worked example: two routes, each the other's check

Solve $x + 2y = 8$, $3x + y = 9$.

- Elimination: $\text{row2} - 3 \cdot \text{row1} \rightarrow -5y = -15$, so $y = 3$, back-substitute $x = 2$.
- Inverse: $\det = 1 \cdot 1 - 2 \cdot 3 = -5$; $A^{-1} = (1/-5)(1-2; -31)$, so $x = (8-2 \cdot 9)/(-5) = 2$, $y = (-24 + 9)/(-5) = 3$.

Substitution closes it: $2 + 2(3) = 8 \checkmark$, $3(2) + 3 = 9 \checkmark$. The **matrix inverse** route costs more arithmetic but verifies the elimination result with an independent method.

■ Scaling intuition for later courses

Nodal analysis IS $A\mathbf{x} = \mathbf{b}$: A holds the conductances, $\mathbf{x}$ the node voltages, $\mathbf{b}$ the source currents. A well-posed circuit gives a non-singular A ; a singular one means the modelling missed a ground or duplicated a source constraint – the determinant is a circuit-debugging tool, not just algebra.

2 Practice

2.1 Solve by elimination: $2x + y = 7$, $x - y = 2$. Verify by substitution. [3]

2.2 Compute the determinant of $\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ and use the inverse to solve $x + 2y = 8$, $3x + y = 9$. [4]

2.3 For which value of k is $\begin{pmatrix} 2 & k \\ 4 & 6 \end{pmatrix}$ singular? Explain the geometric meaning in one sentence. [3]

3 Exam-style questions

3.1 A two-node circuit yields the system $3V_1 - V_2 = 12$, $-V_1 + 4V_2 = 0$ (conductances in siemens, currents in amps).

Solve for V_1 and V_2 by elimination, then verify by the determinant/inverse route. Finally compute the determinant of the coefficient matrix and state what it would mean for the circuit if it were zero. [7]

3.2 Mesh analysis of a two-loop network gives $5I_1 - 3I_2 = 12$ and $-3I_1 + 4I_2 = -6$.

Write the system in matrix form, compute the determinant, solve for both mesh currents, and check both original equations by substitution. State the physical meaning of $I_1 - I_2$. [6]
