

2.1 Calculus review for signals

Name: _____ Class: _____ Date: _____

Total: 22 marks

KEY WORDS derivative • integral • rectangular pulse

Objective

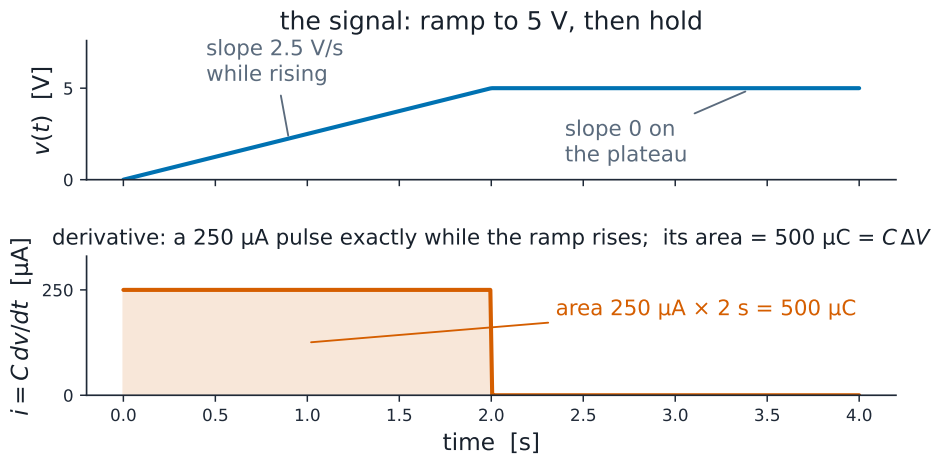
Read derivatives and integrals directly off signals, use the exponential's self-similarity, and check every calculus answer by a second method (area, units, or a capacitor identity).

1 Key knowledge and worked examples

■ Slope and area, on signals

The **derivative** dv/dt is the slope; the **integral** $\int idt$ is the accumulated area. Three facts do most of the EE work:

- $d/dt(e^{at}) = a \cdot e^{at}$ – exponentials are their own slopes.
- A **rectangular pulse** of height A , width τ has area $A\tau$.
- A triangle's area is half its enclosing rectangle.



■ Worked example: the ramp-charged capacitor

$v(t) = 2.5t$ volts for 2 s, $C = 100\mu F$:

- Current: $i = Cdv/dt = 100\mu \times 2.5 = 250\mu A$ – constant, because the ramp's slope is constant.
- Charge: $\int idt = 250\mu \times 2 = 500\mu C$.
- The check: $Q = C \cdot \Delta V = 100\mu \times 5 = 500\mu C$ – the capacitor identity closes the loop with a completely different method.

Two routes, one answer: slope-plus-area against the device law. That habit is this unit.

■ Units are a free check

$[C dv/dt] = \text{farads} \times \text{volts/second} = \text{coulombs/second} = \text{amperes} \checkmark$. An integral over time carries "per second" $\times$ "seconds" = the plain unit – so $\int idt$ in coulombs passes, while a result in amperes would convict the working. Every EE formula can be tried this way in ten seconds.

2 Practice

2.1 Differentiate each: $v = 5t^3$, $i = 4e^{-2t}$, $p = 7$ (constant). [3]

2.2 Compute $\int_0^3 (2t)dt$ and state its units if $2t$ is a current in milliamps with t in seconds. [3]

2.3 A voltage ramps $0 \rightarrow 12$ V in 4 s into a 220 μF capacitor. Find dv/dt , the current, and verify the charge two ways. [4]

3 Exam-style questions

3.1 A current pulse $i(t) = 50 \text{ mA}$ flows for 3 ms into a capacitor, then stops. The capacitor is $47 \text{ }\mu\text{F}$.

Find the charge delivered and the voltage change (two methods for the charge: integral and pulse area). Then compute the capacitor's stored-energy change and check its units reduce to joules. [6]

3.2 A signal $v(t) = 8e^{-3t} \text{ V}$ for $t \geq 0$ decays through a resistor, so $i = v/R$ with $R = 2k\Omega$. Write $i(t)$, evaluate $i(0)$ and $i(0.5)$, and compute the total charge $\int_0^\infty i dt$ (use $\int e^{-at} dt = 1/a$). Verify the total charge also equals the initial current divided by the decay rate. [6]
