12.6 Multirate processing

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS decimation • polyphase

Objective

Order decimation and interpolation correctly, size the protective low-passes, and account polyphase savings.

1 Key knowledge and worked examples

■ The two fences

Decimation by M: low-pass below $f_s/2M$ BEFORE dropping samples. Interpolation by L: stuff $L-1$ zeros, then low-pass below the old Nyquist to kill the images.

■ Worked example: 8 to 48 kHz

Interpolate by 6, low-pass at 4 kHz: images at 8k, 16k, 40k die ✓. Naively the filter runs at 48 kHz – six times the work; polyphase runs 6 subfilters at 8 kHz: same arithmetic, 1/6 the multiplies ✓.

■ Rational ratios

L/M cascades: 48 → 72 kHz is $L=3$, $M=2$ – upsample to 144 kHz, ONE low-pass at 24 kHz stops the ↑ 3 images and the ↓ 2 aliases, then keep every second sample.

2 Practice

2.1 Decimate 48 kHz audio by $M=2$: cutoff of the protective low-pass? [3]

2.2 Upsample 8 kHz speech by 3: at what frequencies do the first two images sit? [3]

2.3 Resample 44.1 kHz to 48 kHz – give a workable L and M. [3]

3 Exam-style questions

3.1 A 32 kHz system must deliver a 8 kHz output.

State the two stages and the order of filtering. Give the protective cutoff and the reason it cannot be skipped. Compute the output sample rate. Explain what changes if the low-pass is applied AFTER the decimation instead. [7]

3.2 An audio path upsamples 8 kHz speech to 48 kHz with a 200-tap low-pass.

Compute the multiplies per second running naively at 48 kHz. Compute the polyphase arrangement's rate (6 subfilters at 8 kHz) and its multiplies per second. State the gain factor missing when zero-stuffing (and where to put it), and explain why polyphase is exact, not approximate. [8]