

12.3 The FFT

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS FFT • butterfly

Objective

Count FFT operations, map bins to hertz, and check real-time feasibility with operation counts.

1 Key knowledge and worked examples

■ **The butterfly**

Even/odd split + twiddle factors: $\log_2 N$ stages of $N/2$ butterflies. Counting complex operations: direct N^2 , FFT $N \log_2 N$ – the gap widens with N .

■ **Worked example: the count and the clock**

$N = 1024$: $N^2 = 1\,048\,576$ against $N \log_2 N = 10\,240$ – $102\times$ fewer ✓. At 10 ns per operation the transform takes about $100\ \mu\text{s}$ – real time against a 1.024 s record.

■ **Bin to hertz**

Bin k sits at $f = k \cdot f_s / N$. Bin 100 of the $N = 1024$, $f_s = 1\ \text{kHz}$ transform is 97.66 Hz ✓ – reading bin numbers as hertz is the classic lab error.

2 Practice

2.1 How many stages and butterflies for an $N = 64$ radix-2 FFT? [3]

2.2 At $f_s = 8\ \text{kHz}$ with $N = 512$: bin width, and the frequency of bin 90. [3]

2.3 Direct DFT of $N = 256$: operation count, and the FFT count. [3]

3 Exam-style questions

3.1 A real-time analyser samples at $f_s = 48\ \text{kHz}$ and must update every 21.3 ms (1024-sample hop).

Compute the bin width. Compute the FFT operation count and, at 5 ns per operation, the compute time per transform; state the margin against the hop. Compute which bin a 1 kHz tone lands in (and its exact frequency). [8]

3.2 A colleague proposes replacing an $N = 1024$ FFT by two $N = 512$ FFTs plus 512 combining butterflies to halve the latency.

Check the operation count of the proposal against one $N = 1024$ FFT. Explain where the saving (if any) comes from, and what Danielson–Lanczos says the structure actually is. State one reason the split helps pipeline implementations. [7]