UNIVERSITY ELECTRICAL ENGINEERING • EXERCISE SHEET

12.2 DTFT and DFT

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS DFT • leakage

Objective

Compute small DFTs by hand, predict leakage from window choice, and turn resolution requirements into record lengths.

1 Key knowledge and worked examples

■ From DTFT to DFT

$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}$ – the DTFT sampled at N points; sampling in frequency makes the time signal periodic, which is why off-bin tones leak.

■ Worked example: on-bin versus off-bin

8-point DFT of a cosine on bin 1: $|X| = 4$ at bins 1 and 7, zero elsewhere ✓. Half a bin off ($1.5 \cdot f_s/8$): magnitudes 2.40, 2.80, 1.19, 1.0 – energy in every bin. The rectangular window's −13 dB sidelobes are speaking; Hamming drops them to −43 dB.

■ The resolution budget

$\Delta f = f_s/N$: at $f_s = 1$ kHz, $N = 100$ gives 10 Hz bins, $N = 1000$ gives 1 Hz. Zero-padding interpolates the same spectrum – it resolves nothing new.

2 Practice

2.1 At $f_s = 2$ kHz, what N gives 4 Hz bins? [3]

2.2 A tone sits at bin 3 of an on-bin 8-point DFT with amplitude 2. Give $|X[k]|$ for all bins. [3]

2.3 Two tones 3 Hz apart must be resolved at $f_s = 1$ kHz. Minimum N, and why padding to 8192 does not help. [3]

3 Exam-style questions

3.1 A spectrum analyser at $f_s = 1$ kHz must resolve the 50 Hz mains tone from a 55 Hz interferer.

State the minimum N and the record length in seconds. Compute the bin width for $N = 256$ and state whether the two tones are resolvable. Explain what zero-padding to 2048 changes and what it cannot change. [7]

3.2 An 8-point DFT of $x[n] = \cos(2\pi \times 1.5 \times n/8)$ is computed with a rectangular window.

Explain why every bin is non-zero. Give the two largest bins and their magnitudes. The same tone is now windowed with Hamming: state what happens to the sidelobe level and to the main-lobe width, and the consequence for resolving two nearby tones. [8]