

12.1 Discrete-time signals and systems review

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS impulse response • convolution

Objective

Compute discrete convolutions by hand, read stability and causality off an impulse response, and connect poles to settling behaviour.

1 Key knowledge and worked examples

■ The sliding table

$y[n] = \sum_k h[k] \cdot x[n - k]$: slide the taps, add one diagonal at a time. A step through any FIR settles at $\sum h$ – the DC gain $H(1)$.

■ Worked example: two smoothers

$h = [1, 2, 1]$ on a unit step: $y = 1, 3, 4, 4, \dots$ ✓ settles at 4. The recursive averager $y[n] = (y[n - 1] + x[n])/2$ steps up 0.5, 0.75, 0.875, 0.9375 = $1 - 2^{-n}$ – its pole sits at $z = 0.5$, so it approaches but never arrives.

■ Stability and causality

Causality: $h[n] = 0$ for $n < 0$. BIBO stability: $\sum |h[n]|$ converges – for rational systems, all poles inside the unit circle.

2 Practice

2.1 Compute $y[n]$ for $h = [1, 3, 2]$ and $x = [1, 1, 1]$. [3]

2.2 A filter has $h[n] = 0.4^n$ for $n \geq 0$. Is it BIBO stable? Justify with the sum. [3]

2.3 Give the DC gain of $h = [2, -1, 2]$ and check it against the steady step output. [3]

3 Exam-style questions

3.1 A 3-tap FIR has $h = [1, a, 1]$.

For $a = 2$ compute the step response and its settling value. For which a does the step response stay non-negative at every sample? State the causality and BIBO status of this filter, with reasons. [7]

3.2 The leaky integrator $y[n] = \alpha y[n-1] + x[n]$ with $y[-1] = 0$ and $0 < \alpha < 1$.

Compute the impulse response, the step response at $n = 0, 1, 2, 3$ for $\alpha = 0.5$, and its closed form. State the pole, the stability condition on α , and how many samples until the step response is within 1% of its limit when $\alpha = 0.5$. [8]
