

11.7 Information and coding introduction

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS entropy • channel capacity • Hamming code

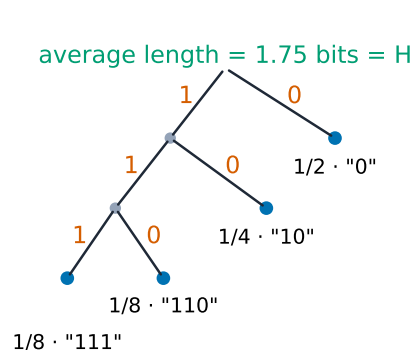
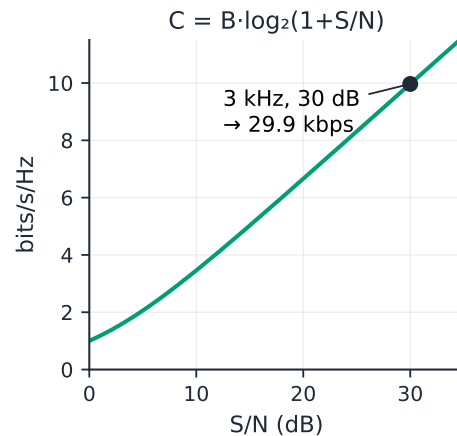
Objective

Compute entropy and redundancy, state channel capacity as a hard limit, and apply parity and Hamming codes to detect and correct errors.

1 Key knowledge and worked examples

■ Entropy

$H = -\sum p \cdot \log_2 p$ bits/symbol: the quartile source (1/2, 1/4, 1/8, 1/8) gives $H = 1.75$ against 2 fixed – Huffman lengths 1,2,3,3 average exactly 1.75 ✓ (dyadic probabilities meet the bound).



Optimal lengths average exactly the entropy for dyadic probabilities; capacity is approached, never beaten.

■ Capacity

$C = B \cdot \log_2(1 + S/N)$: 3 kHz at 30 dB → 29.9 kbps – the dial-up ceiling, approached (V.90 got close) but never beaten.

■ Hamming (7,4)

4 data + 3 parity, distance 3: any single error is CORRECTED – encode 1011 → 0 1 1 0 0 1 1; flip bit 5 and the syndrome ($s_3s_2s_1$ read as binary) points at 5 exactly ✓.

2 Practice

2.1 Compute the entropy of a uniform 4-symbol source. [3]

2.2 Compute the average Huffman codeword length for (0.4, 0.3, 0.2, 0.1) with lengths 1, 2, 3, 3. [3]

2.3 State the rate of Hamming (7,4) and how many errors it corrects. [3]

3 Exam-style questions

3.1 A source emits symbols with probabilities 0.4, 0.3, 0.2, 0.1.

Compute the entropy. Build a Huffman code (show the merging order) and compute its average length. Compute the code efficiency against entropy and the redundancy of the 2-bit fixed code. State why the Huffman average exceeds entropy slightly for this source. [8]

3.2 Hamming (7,4) protects a link; the receiver computes the syndrome from the received 7-bit word.

Give the parity-check structure (which bit positions each of s_1 , s_2 , s_3 covers). Encode the data word 0110. Flip bit 3 (counting from 1) in the codeword and compute the syndrome, showing that it identifies the error position. State what happens for a DOUBLE error and how the extended (8,4) code (distance 4) changes that. [7]
